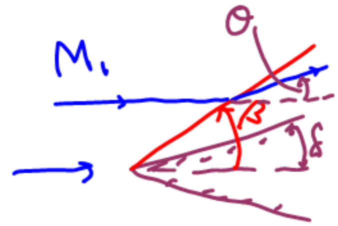
Today, we will:

- Continue to derive the equations for 2-D oblique shocks; in particular, generate and discuss **the θ-β-M relationship**
- Provide some qualitative comments about the θ-β-M relationship

Recall the geometry of an oblique shock from the previous lecture:



β = oblique shock angle

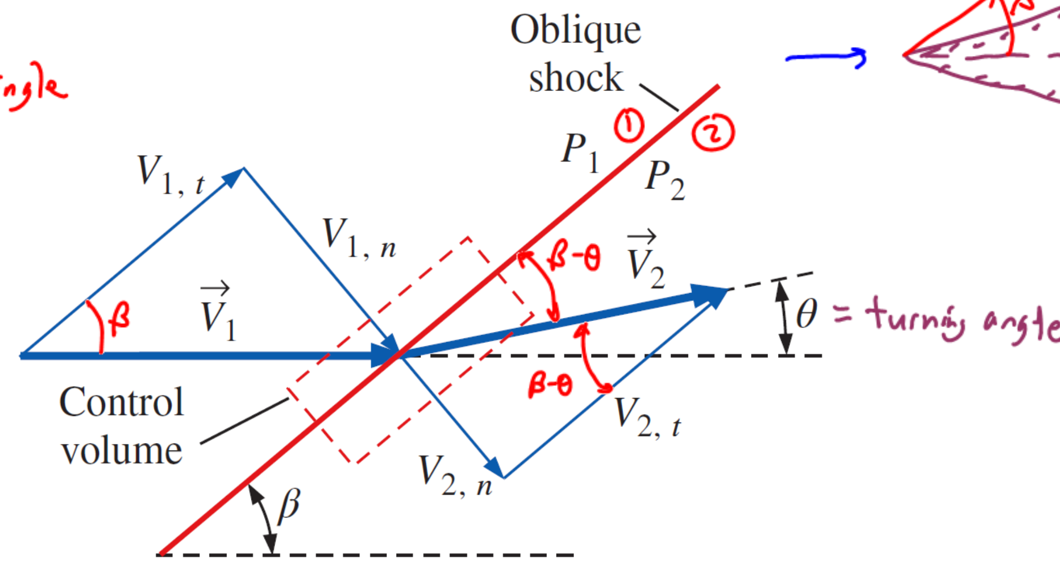


Figure from Çengel and Cimbala, Ed. 4.

HOW ARE θ, β, & M_1 RELATED?

ANS: THE **θ-β-M relationship**

FROM THE DIAGRAM, $\boxed{V_{1,t} = \dfrac{V_{1,n}}{\tan\beta}}$ $\boxed{V_{2,t} = \dfrac{V_{2,n}}{\tan(\beta-\theta)}}$

BUT $V_{1,t} = V_{2,t}$ equate these

$$\boxed{\frac{V_{1,n}}{V_{2,n}} = \frac{\tan(\beta)}{\tan(\beta-\theta)}} \quad (6)$$

Now recall our normal shock relations: $\boxed{\dfrac{\rho_2}{\rho_1} = \dfrac{V_1}{V_2} = \dfrac{(\gamma+1)M_1^2}{2+(\gamma-1)M_1^2}}$

PLUG IN $\boxed{M_{1,n} = M_1 \sin\beta}$

So, the normal component eq. for our oblique shock becomes

$$\frac{V_{1,n}}{V_{2,n}} = \frac{(\gamma+1) M_{1,n}^2}{2+(\gamma-1) M_{1,n}^2} = \boxed{\frac{(\gamma+1) M_1^2 \sin^2\beta}{2+(\gamma-1) M_1^2 \sin^2\beta} = \frac{V_{1,n}}{V_{2,n}}} \quad (7)$$

EQUATE (6) & (7):

$$\boxed{\frac{\tan\beta}{\tan(\beta-\theta)} = \frac{(\gamma+1) M_1^2 \sin^2\beta}{2+(\gamma-1) M_1^2 \sin^2\beta}} \quad (8)$$

☆ TRIG IDENTITIES

$$\boxed{\tan\theta = \frac{\sin\theta}{\cos\theta}}$$

$$\boxed{\cot\beta = \frac{1}{\tan\beta}}$$

$$\boxed{\cos(2\beta) = \cos^2\beta - \sin^2\beta}$$

$$\boxed{\tan(\beta-\theta) = \frac{\tan\beta - \tan\theta}{1+\tan\beta\tan\theta}}$$

(cold rainy night)

OUR "workhorse eq!" for oblique shock analysis ☆☆☆

$$\boxed{\tan\theta = \frac{2\cot\beta\,(M_1^2 \sin^2\beta - 1)}{M_1^2(\gamma + \cos 2\beta) + 2}} \quad (9)$$

THE θ-β-M EQUATION FOR OBLIQUE SHOCKS ☆

(For ideal gases) ☆

COMMENTS: • θ is a unique function of M & β

$$\boxed{\theta = \tan^{-1}\left[\frac{2\cot\beta\,(M_1^2\sin^2\beta - 1)}{M_1^2(\gamma + \cos 2\beta) + 2}\right]} \quad * (9)$$

• This eq is EXPLICIT for solving for θ

• We can plot θ as a fnc of β & M_1

THE θ-β-M PLOT

HOW TO CREATE THIS PLOT:

• Pick an M_1 (e.g., $M_1 = 1.1$)

• Create a range of β

• Use Eq (9) to calc θ @ each of these β's

• Plot θ vs β @ this M_1

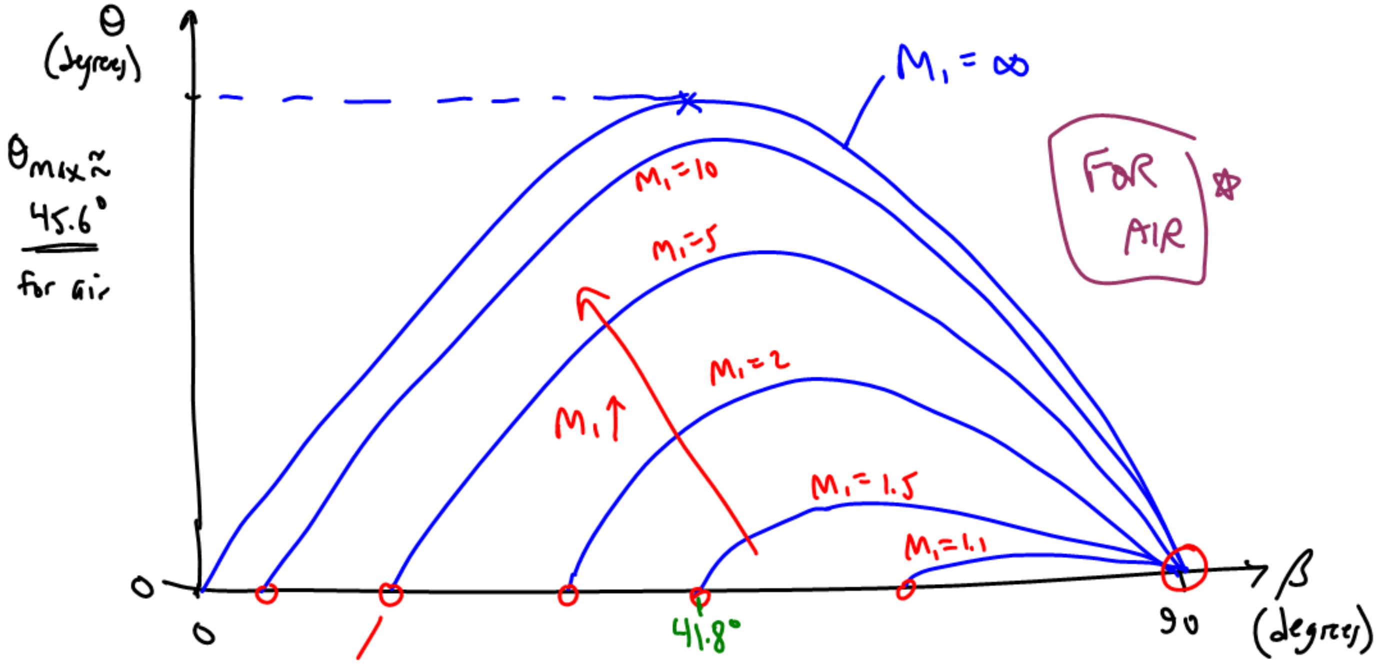
• REPEAT FOR OTHER VALUES OF M_1

↓

THE θ-β-M PLOT !

Start @ 90° = NORMAL SHOCK

β	θ
90°	—
89.8°	—
89.6°	—
⋮ ↓	— ↓



MINIMUM β = WEAKEST POSSIBLE OBLIQUE SHOCK AT A GIVEN M_1 $(\theta = 0)$

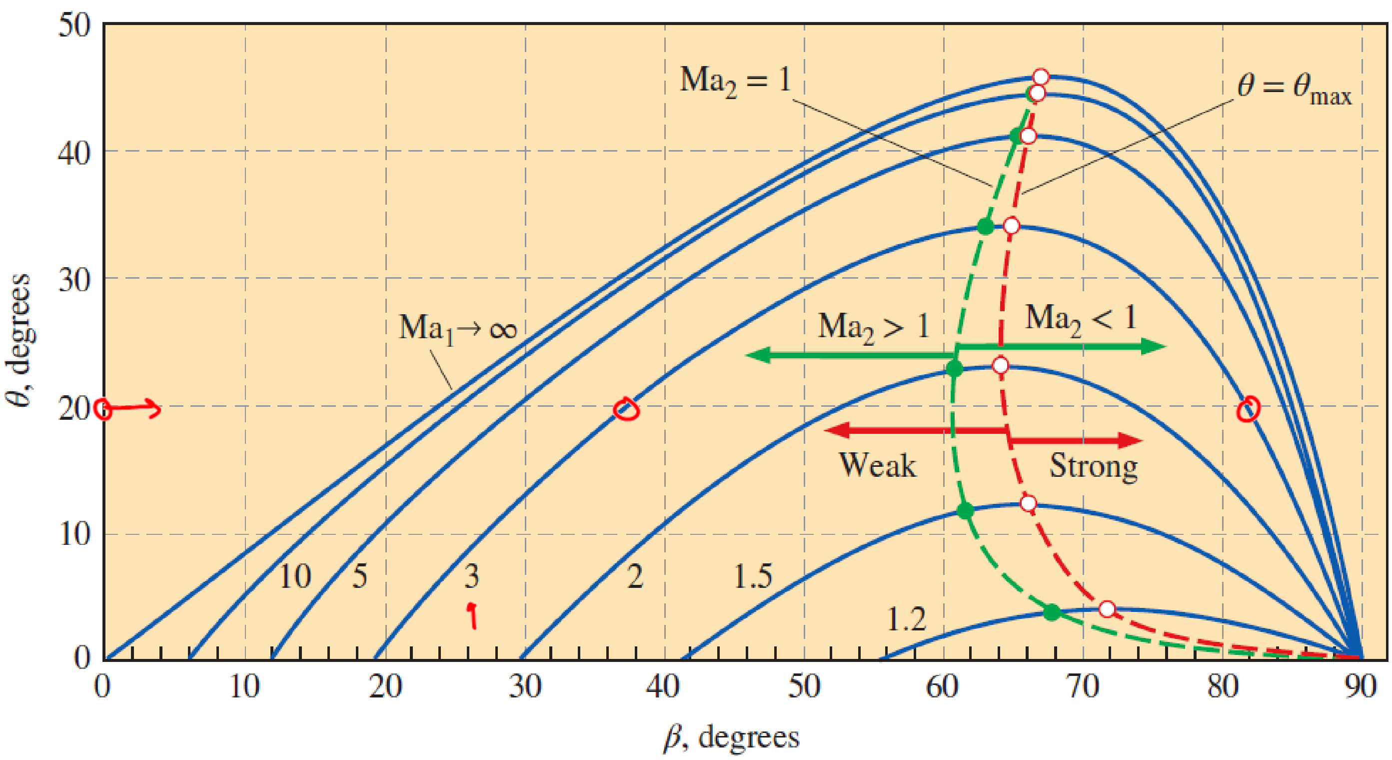
THIS IS A MACH WAVE ! $\boxed{\beta = \mu = \sin^{-1}\frac{1}{M_1}}$ ✡

β_{min}

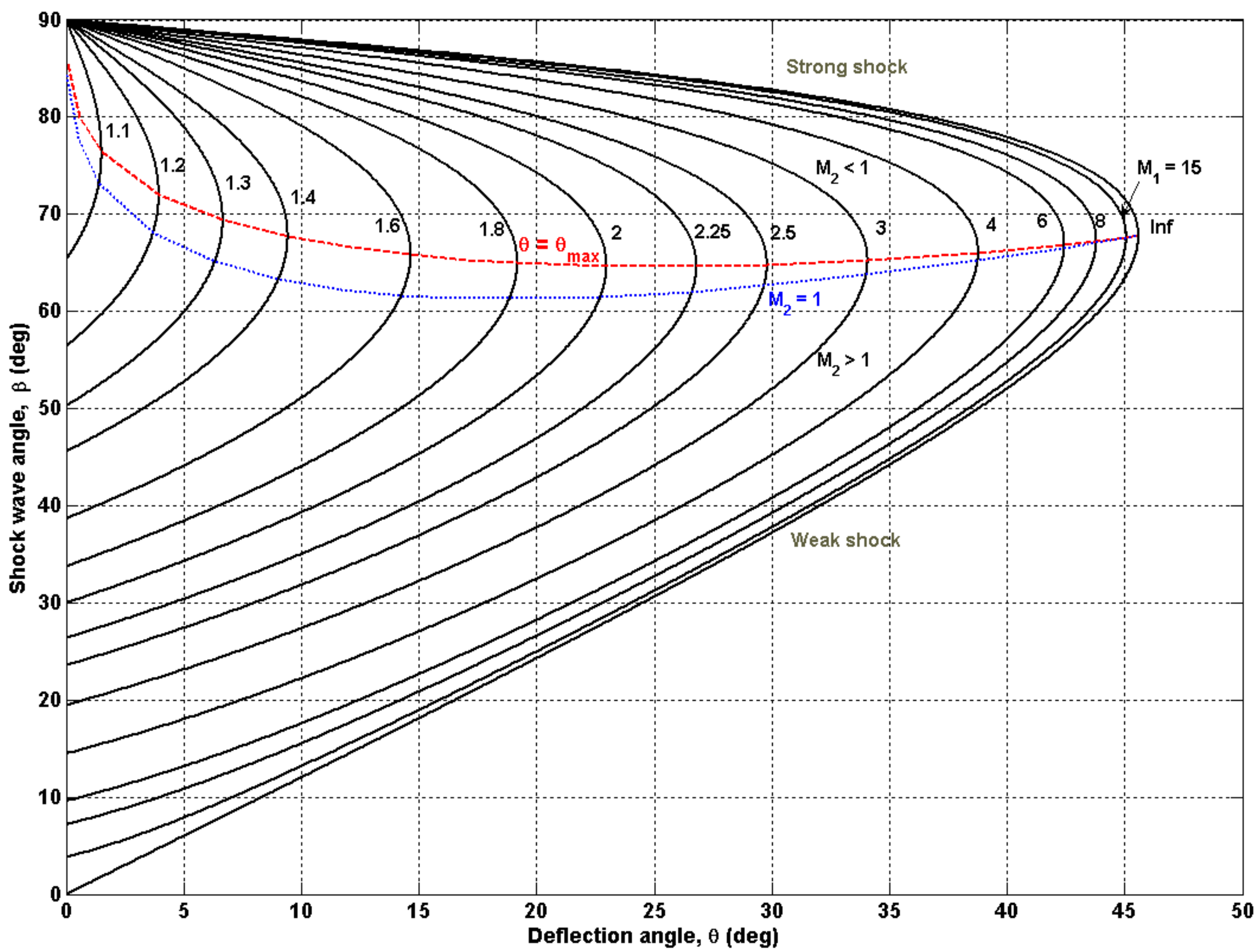
E.g., @ $M_1 = 1.50$, $\mu = 41.8°$

IT IS IMPOSSIBLE FOR β TO BE SMALLER THAN β_{min} for a given M_1 & γ

From my fluids book:

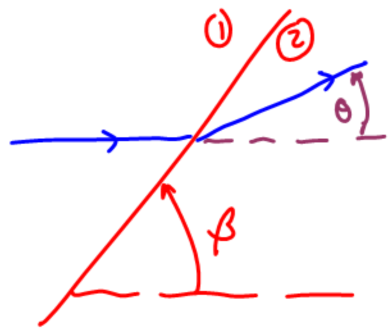


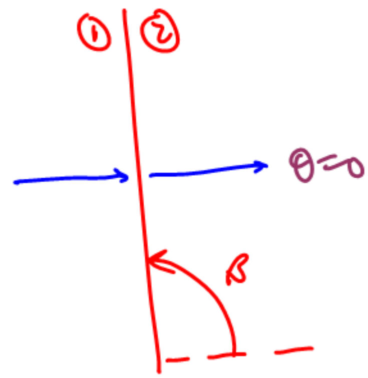
Some authors flip the axes:



COMMENTS ABOUT THE θ-β-M PLOT

- THERE IS A UNIQUE CURVE FOR ANY $M_1 > 1$ $(1 < M_1 < \infty)$
- ALL CURVES INTERSECT AT $\theta = 0$, $\beta = 90^\circ$ → THIS IS THE STRONGEST POSSIBLE SHOCK!



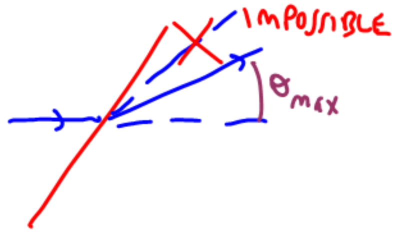


- FOR A GIVEN M_1, AT ANY $\beta < 90^\circ$, THE SHOCK IS **WEAKER** THAN THE NORMAL SHOCK
- β CANNOT GO TO 0° UNLESS $M_1 \to \infty$
- $\beta_{min} \downarrow$ AS $M_1 \uparrow$

- FOR A GIVEN M_1
  - $\theta = 0$ @ some minimum $\beta = \beta_{min} = \mu$ = Mach angle → Isentropic Mach wave
  - $\theta = 0$ @ $\beta = 90^\circ$ → NORMAL SHOCK
  - For $\mu < \beta < 90^\circ$, θ reaches a maximum @ some β
    ↓
    θ_{max}
- $\theta_{max} \uparrow$ as $M_1 \uparrow$ (see red line on the plot)

- ABOVE θ_{max}, AN OBLIQUE SHOCK CANNOT EXIST !

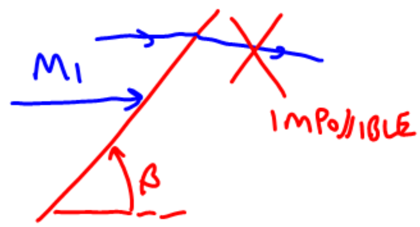
(THE FLOW CANNOT TURN THAT SHARPLY)



- BELOW β_{min}, AN OBLIQUE SHOCK CANNOT EXIST AT THE ASSOCIATED M_1

- MATHEMATICALLY, EQ (9) CAN YIELD $\theta < 0$

★ IGNORE THESE



- THE θ-β-M EQ IS EXPLICIT FOR θ

(θ is a unique value for a given β, M_1 & γ)

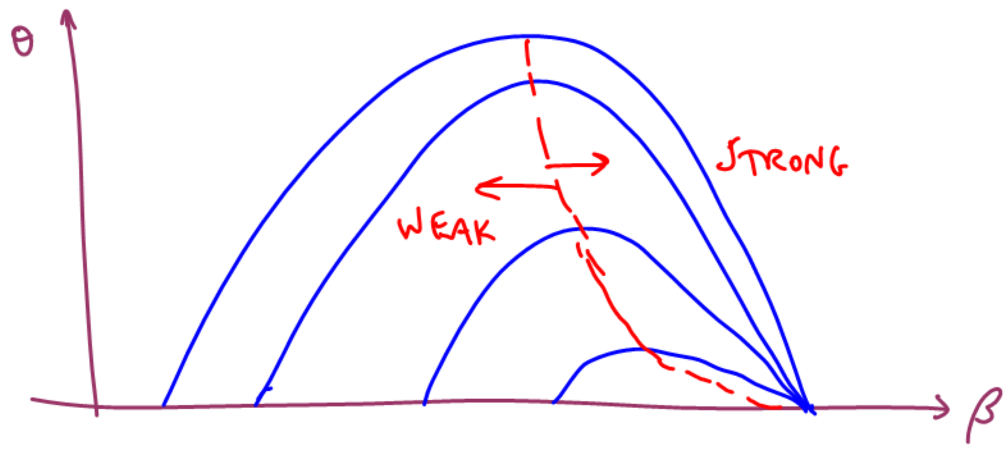
- UNFORTUNATELY, WE USUALLY KNOW θ & M_1 & WANT TO PREDICT β

BUT THE θ-β-M EQ IS IMPLICIT FOR

$$\boxed{\beta = \text{fnc}(\theta, M_1, \gamma)}$$

ITERATION IS NECESSARY

- FOR A GIVEN θ $(\theta < \theta_{max})$ & M_1, THERE ARE TWO POSSIBLE β's !!



Any β to the left of $\theta = \theta_{max}$ is a WEAK OBLIQUE SHOCK

Any β to the right of $\theta = \theta_{max}$ is a STRONG OBLIQUE SHOCK

HOW DO WE KNOW WHICH ONE IS CORRECT?

ANSWER: THE WEAK ONE IS "PREFERRED" & MORE COMMON

THE STRONG ONE MUST HAVE VERY HIGH DOWNSTREAM PRESSURE TO FORM