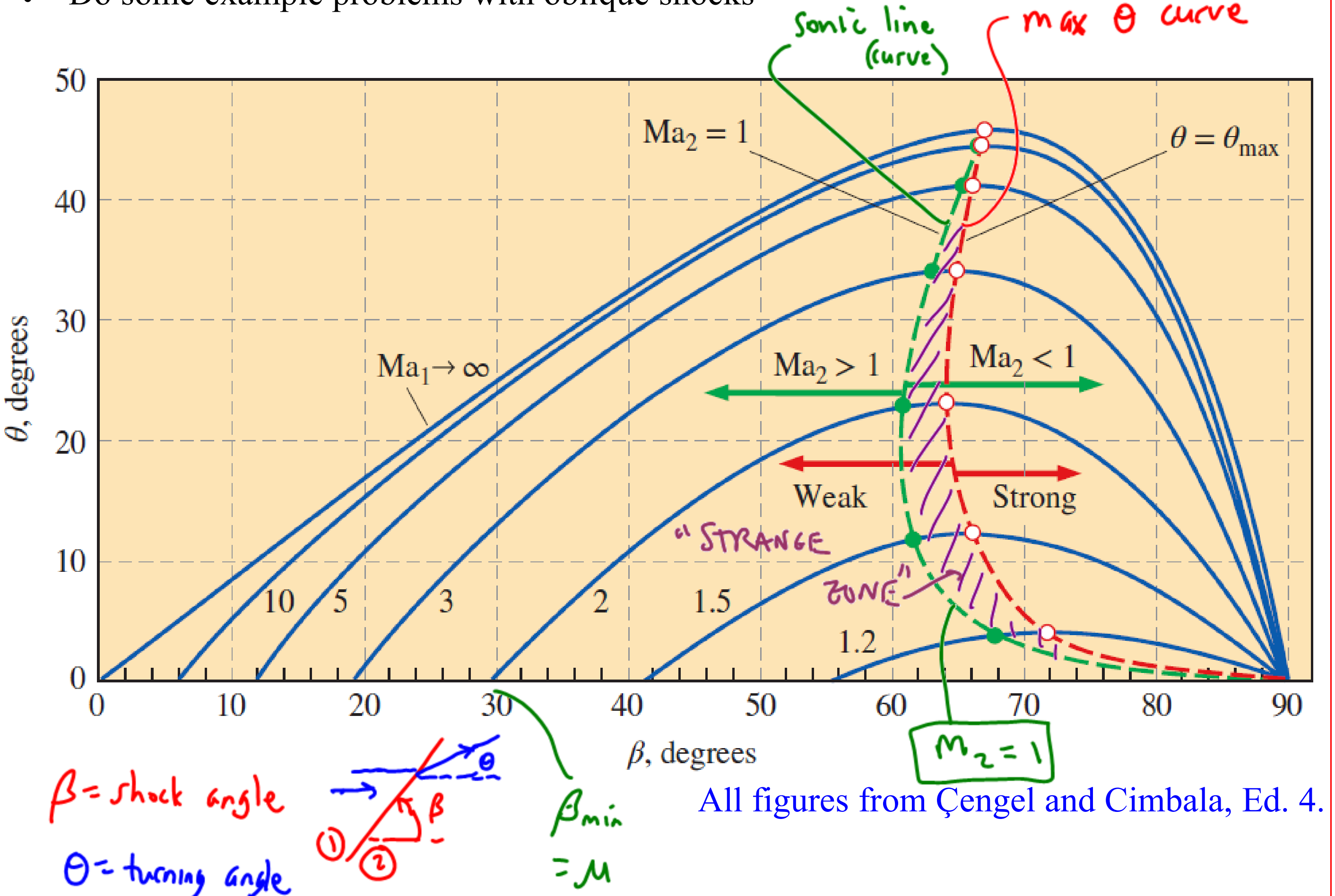
Today, we will:

- Continue discussing oblique shocks: more comments about the θ-β-M plot
- Do some example problems with oblique shocks



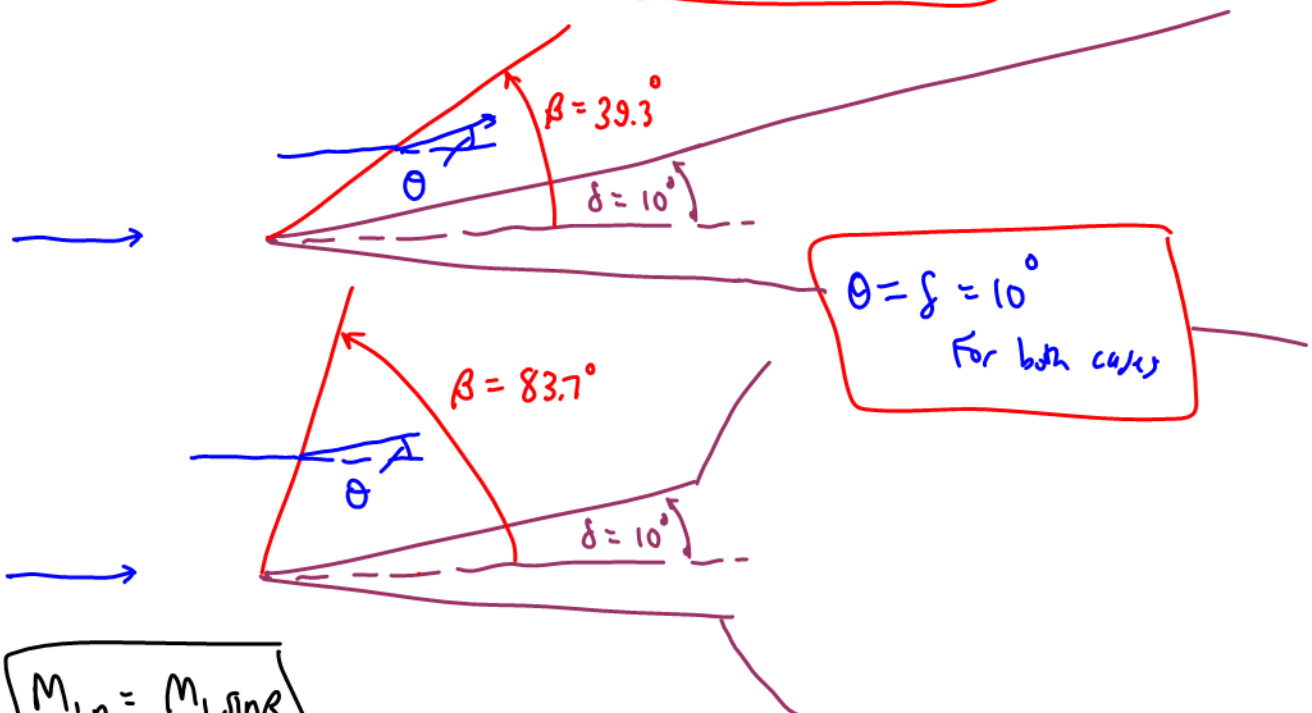
All figures from Çengel and Cimbala, Ed. 4.

- To the left of $M_2=1$, $M_2>1$ since the shock is weak
  ② is supersonic
  (M_2 does not differ much from M_1)
- To the right of $M_2=1$, $M_2<1$ since the shock is stronger
  ② is subsonic
  (M_2 differs a lot from M_1)
- "STRANGE ZONE" → The oblique shock is WEAK, BUT $M_2<1$
  - FOR ALL STRONG OBLIQUE SHOCKS, $M_2<1$
  - FOR MOST WEAK OBLIQUE SHOCKS, $M_2>1$ (except in the strange zone)

EXAMPLE

Air @ $M_1 = 2.00$ turns through an oblique shock with $\theta = 10^\circ$

- USING THE θ-β-M eq → $\beta_{weak} = 39.3^\circ$, $\beta_{strong} = 83.7^\circ$



$$M_{1,n} = M_1 \sin\beta$$

use this instead of M_1 in our previous eq's for a normal shock

Here,

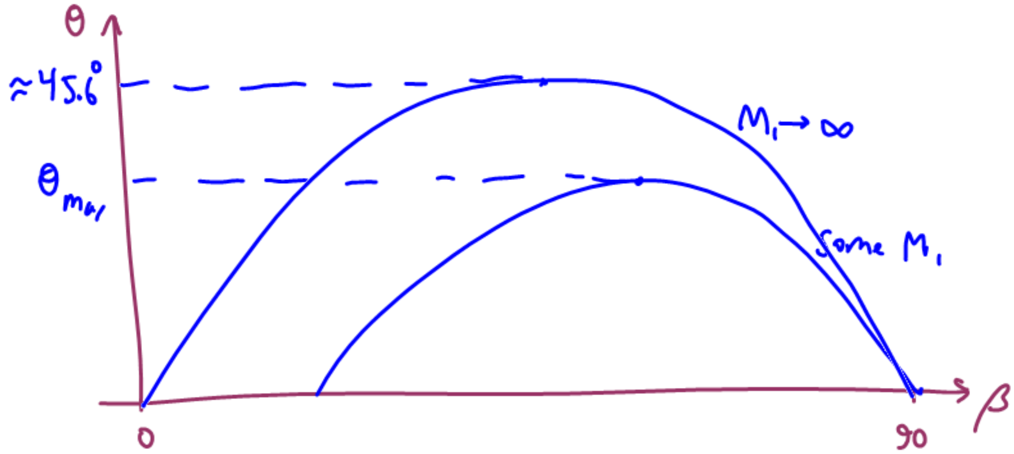
WEAK: $M_{1,n} = 2.00 \sin(39.3^\circ) = \underline{1.267}$ → $M_{2,n} = 0.803$ ← SUBSONIC

STRONG: $M_{1,n} = 2.00 \sin(83.7^\circ) = \underline{1.988}$ → $M_{2,n} = 0.579$

recall, $M_{2,n} = M_2 \sin(\beta - \theta)$

$$\frac{P_2}{P_1} = \frac{2\gamma M_{1,n}^2 - \gamma + 1}{\gamma + 1} = 1.707 \text{ (WEAK)} \;\; \& \;\; 4.444 \text{ (STRONG)}$$

☺



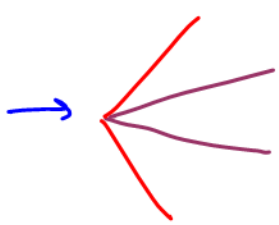
WHAT HAPPENS IF $\delta > \theta_{max}$ FOR A 2-D WEDGE?

☆ IT IS IMPOSSIBLE TO HAVE A STRAIGHT OBLIQUE SHOCK WITH A TURNING ANGLE $> \theta_{max}$ FOR A GIVEN M_1

IN AIR IT IS IMPOSSIBLE TO HAVE A STRAIGHT OBLIQUE SHOCK WITH A TURNING ANGLE $> \approx 45.6°$

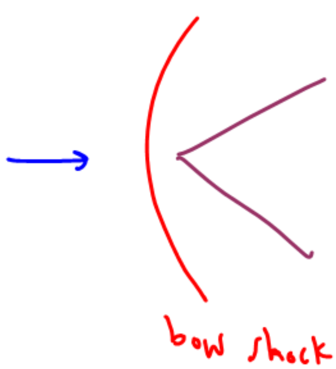
ANSWER: THE SHOCK DETACHES & BECOMES A BOW SHOCK

E.g., @ $M_1 = 2.00$ $\theta_{max} \approx 23.0°$, where $\beta = 64.6°$

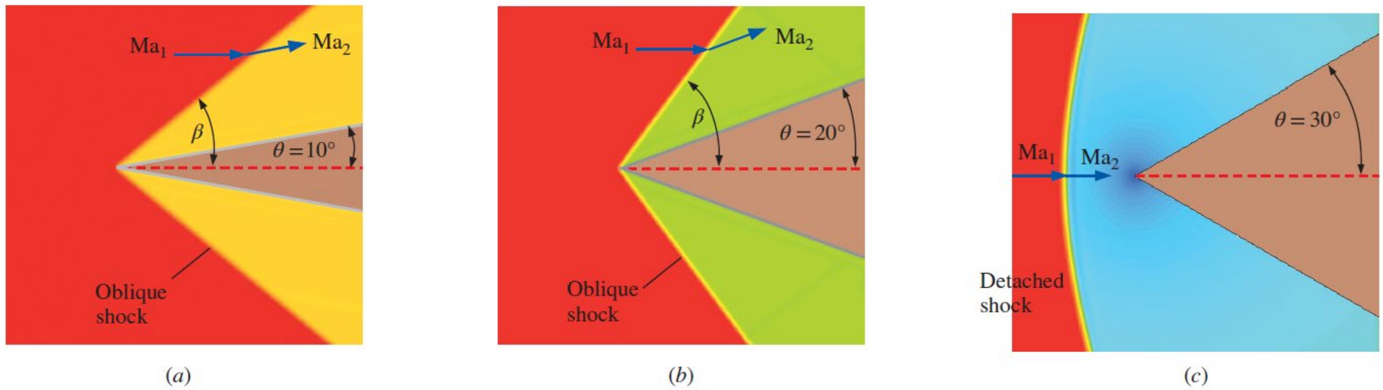


For $\theta < \theta_{max}$, a straight oblique shock is possible

For $\theta > \theta_{max}$, " " " " " not "



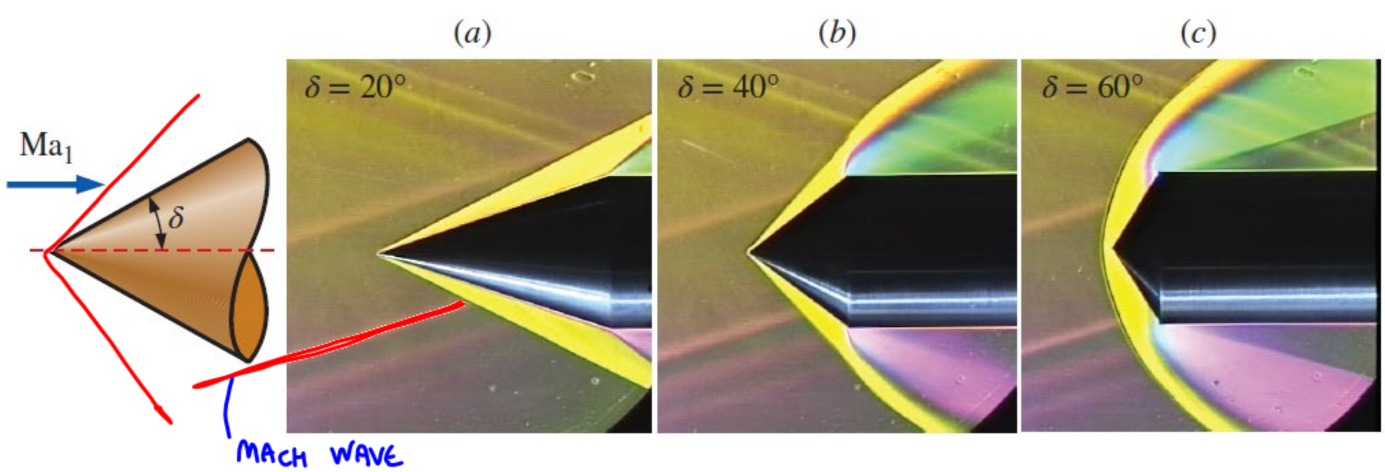
CFD Example – 2-D wedge of various half angles (CFD by J. M. Cimbala):



Steady 2-D Supersonic Flow Over a Wedge: These oblique shocks are formed by supersonic flow impinging on three two-dimensional wedges with different wedge angles: θ = (a) 10°, (b) 20°, and (c) 30°. Upstream Mach number = 2.0. Contours of Mach number are shown, ranging from 0.2 (dark blue) to 2.0 (dark red).

At a given M_1, there is a maximum wedge half angle above which a bow shock is the only valid solution

Experimental Example – Axisymmetric blunt body with conical-nose of various half angles (Experiment by G. S. Settles):



Steady Axisymmetric Supersonic Flow Over a Cone. Similar behavior occurs for the *axisymmetric* case (cones), but the angles are different. Shown are color schlieren images

HERE, MACH WAVE ANGLE = 18.4° ⇒ WHAT IS M_1?

(18.4° = μ)

$$\boxed{\mu = \sin^{-1}\frac{1}{M_1}} \rightarrow M_1 = \frac{1}{\sin\mu} \rightarrow \boxed{M_1 = 3.16}$$

Experimental Example – Spherical body (Experiment by G. S. Settles):

ANY BLUNT-NOSED BODY

WILL HAVE A BOW SHOCK

Steady Axisymmetric Supersonic Flow Over a Sphere. A bow shock forms upstream of *all* such blunt bodies. Shown is a color schlieren image at Mach number 3.0.

THERE IS A DIFFERENT BUT SIMILAR θ-β-M EQ FOR AXISYMMETRIC OBLIQUE SHOCKS

COMPARE: $M_1 = 2.0$, $\theta = 20°$ $\rightarrow$ $\beta = 53.4°$ (weak case) 2-D

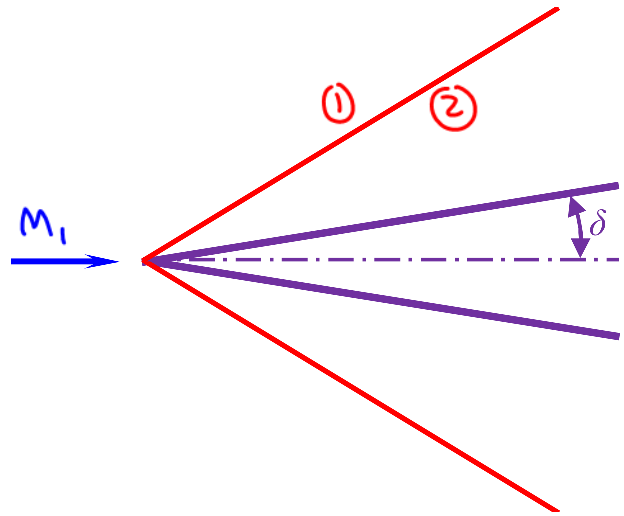
$M_1 = 2.0$, $\theta = 20°$ $\rightarrow$ $\beta = 37.8°$ AXISYM

(no weak/strong axisymmetric cases – just one)

Example: Supersonic Flow Over a 2-D Wedge

Given: Air flowing at supersonic speed strikes a two-dimensional wedge. Here are some values:

- P_1 = 101.325 kPa (standard atmospheric pressure)
- T_1 = 288.0 K
- δ = 20° (wedge half angle)
- M_1 = 3.00 (upstream Mach number)



To do: Calculate shock angle β, air properties P_2 and T_2, and Mach number M_2.

Assumptions and Approximations: Steady, ideal gas, adiabatic, ignore boundary layer effects along the walls.

Solution:

- $\theta = \delta = 20.0°$ (Neglecting BL growth along the wall)
- Use θ-β-M eq. to calc. β IMPLICITLY $\boxed{\beta = 37.76°}$ weak case
- $M_{1,n} = M_1 \sin\beta = 3.00 \sin(37.76°) = $ $M_{1,n} = 1.837$
- Use $M_{1,n}$ in our normal shock eqs in place of M_1

$$\boxed{\frac{P_2}{P_1} = \frac{2\gamma M_{1,n}^2 - \gamma + 1}{\gamma + 1}} = 3.7713$$

$$\therefore P_2 = \frac{P_2}{P_1} P_1 = 3.7713\,(101.325\text{ kPa}) = \boxed{P_2 = 382.1\text{ kPa}}$$

$$M_{2,n} = \sqrt{\frac{1 + \frac{\gamma-1}{2} M_{1,n}^2}{\gamma M_{1,n}^2 - \frac{\gamma-1}{2}}} = 0.60839 \qquad \boxed{M_{2,n} = 0.608}$$

$$T_2 = \frac{T_2}{T_1} T_1 = \boxed{T_2 = 449.2\text{ K}}$$

- $M_{2,n} = M_2 \sin(\beta - \theta)$

$$M_2 = \frac{M_{2,n}}{\sin(\beta-\theta)} = \frac{0.60839}{\sin(37.76-20.0)} = \boxed{1.994 = M_2}$$

(Supersonic)

REPEAT FOR STRONG OBLIQUE SHOCK CASE

θ-β-M eq $\rightarrow$ pick a first guess with a large β (e.g. 80°)

SUMMARY OF MY RESULTS:

$\beta = 82.147°$

$M_{1,n} = 2.972$

$M_{2,n} = 0.4769$

$M_2 = 0.5374$ SUBSONIC, SINCE THIS IS A STRONGER SHOCK

Note: "Typo" error in the video. This should be 0.5394

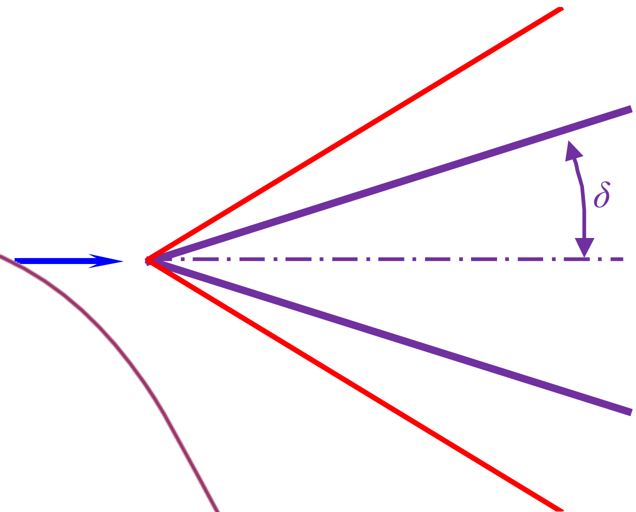
$P_2 = 10.137$ atm

$= 1027$ kPa HIGHER THAN WEAK CASE

$T_2 = 762.1$ K MUCH HIGHER THAN WEAK CASE

Example: Supersonic Flow Over a 2-D Wedge (same as previous example, but with a larger wedge angle)

Given: Air flowing at supersonic speed strikes a two-dimensional wedge. Here are some values:



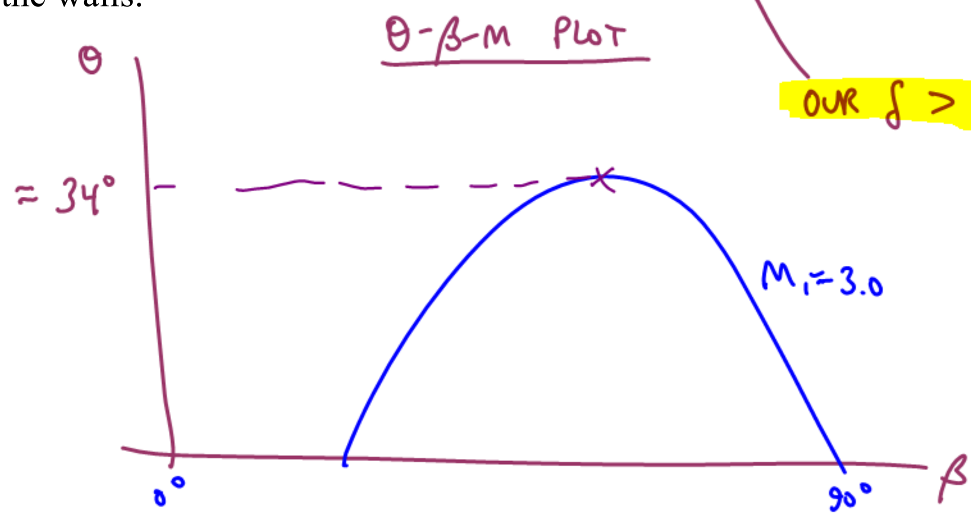
- P_1 = 101.325 kPa (standard atmospheric pressure)
- T_1 = 288.0 K
- δ = 35° (wedge half angle)
- M_1 = 3.00 (upstream Mach number)

To do: Calculate shock angle β, air properties P_2 and T_2, and Mach number M_2.

Assumptions and Approximations: Steady, ideal gas, adiabatic, ignore boundary layer effects along the walls.

Solution:

OUR $\delta > \theta_{MAX}$



AN OBLIQUE SHOCK IS NOT POSSIBLE !

INSTEAD, A **BOW SHOCK** WILL FORM