Sphere Drag ☆

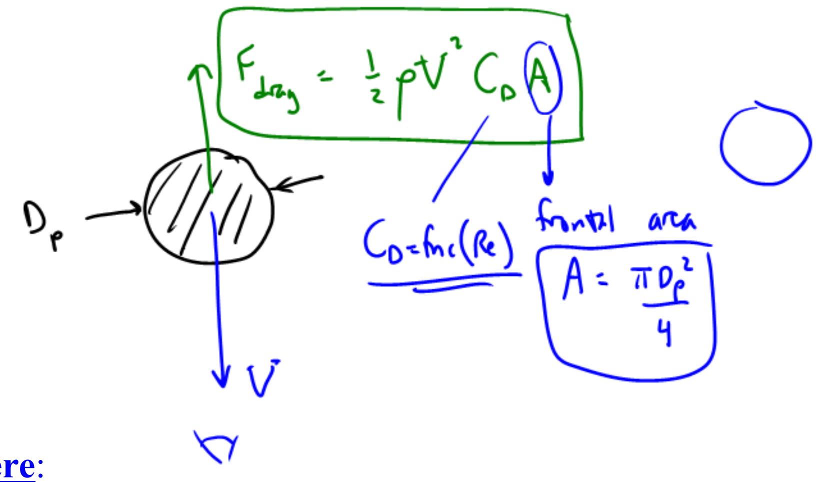
Today, we will:

- Discuss **Drag on Spheres** and various equations for sphere **Drag Coefficient**
- Do an example problem

Drag on Spheres:

$\frac{\mu}{\rho} = \nu$

$$Re = \frac{\rho V D_p}{\mu} = \frac{V D_p}{\nu}$$



Drag coefficient C_D on a sphere:

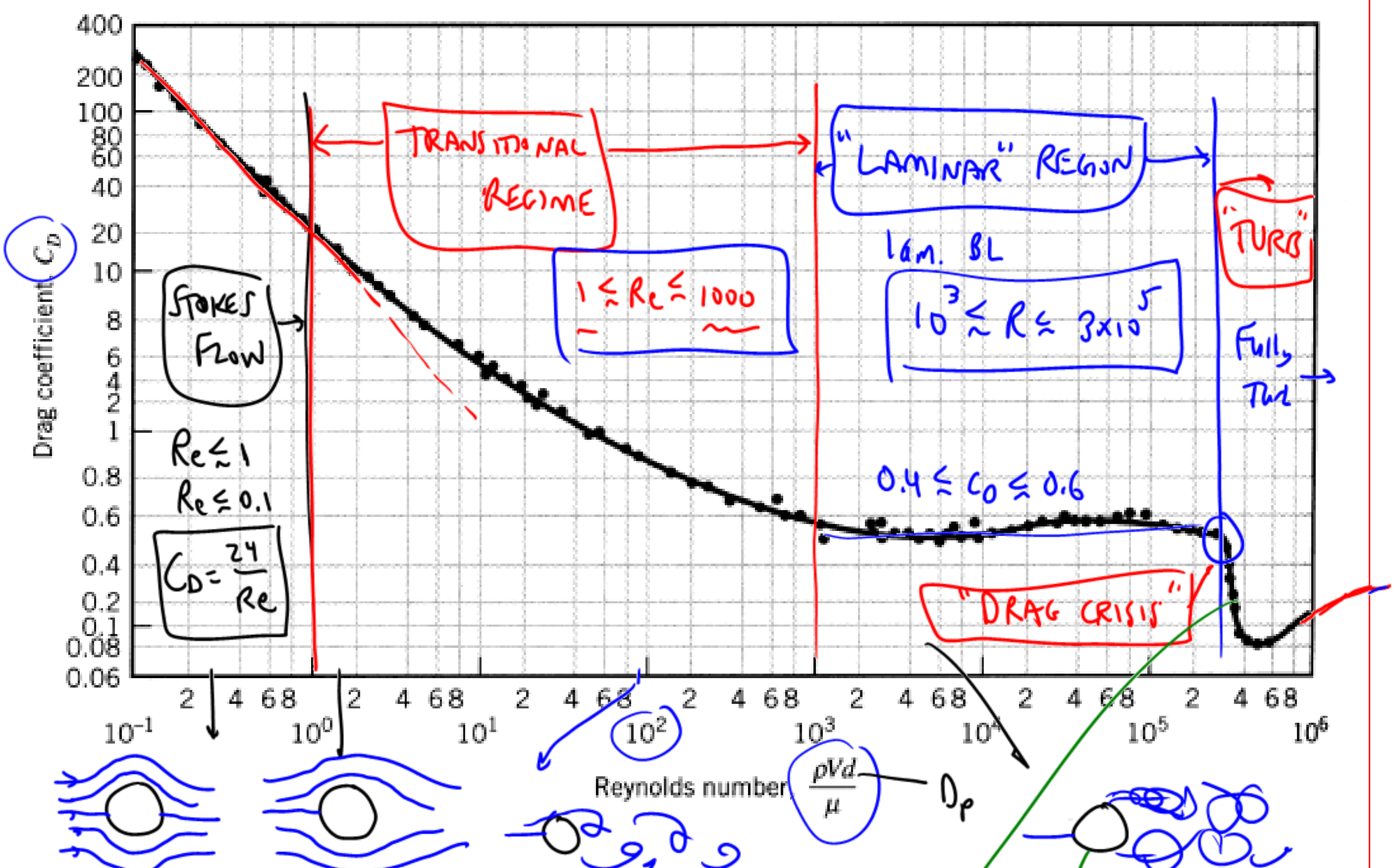
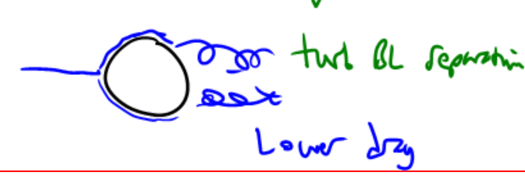


Fig. 2: The effect of Reynolds number on the drag coefficient C_D of a smooth sphere of diameter d.



Some empirical equations for sphere drag coefficient, $C_D = C_D$(Reynolds number):

Stokes flow approx only for very small Re

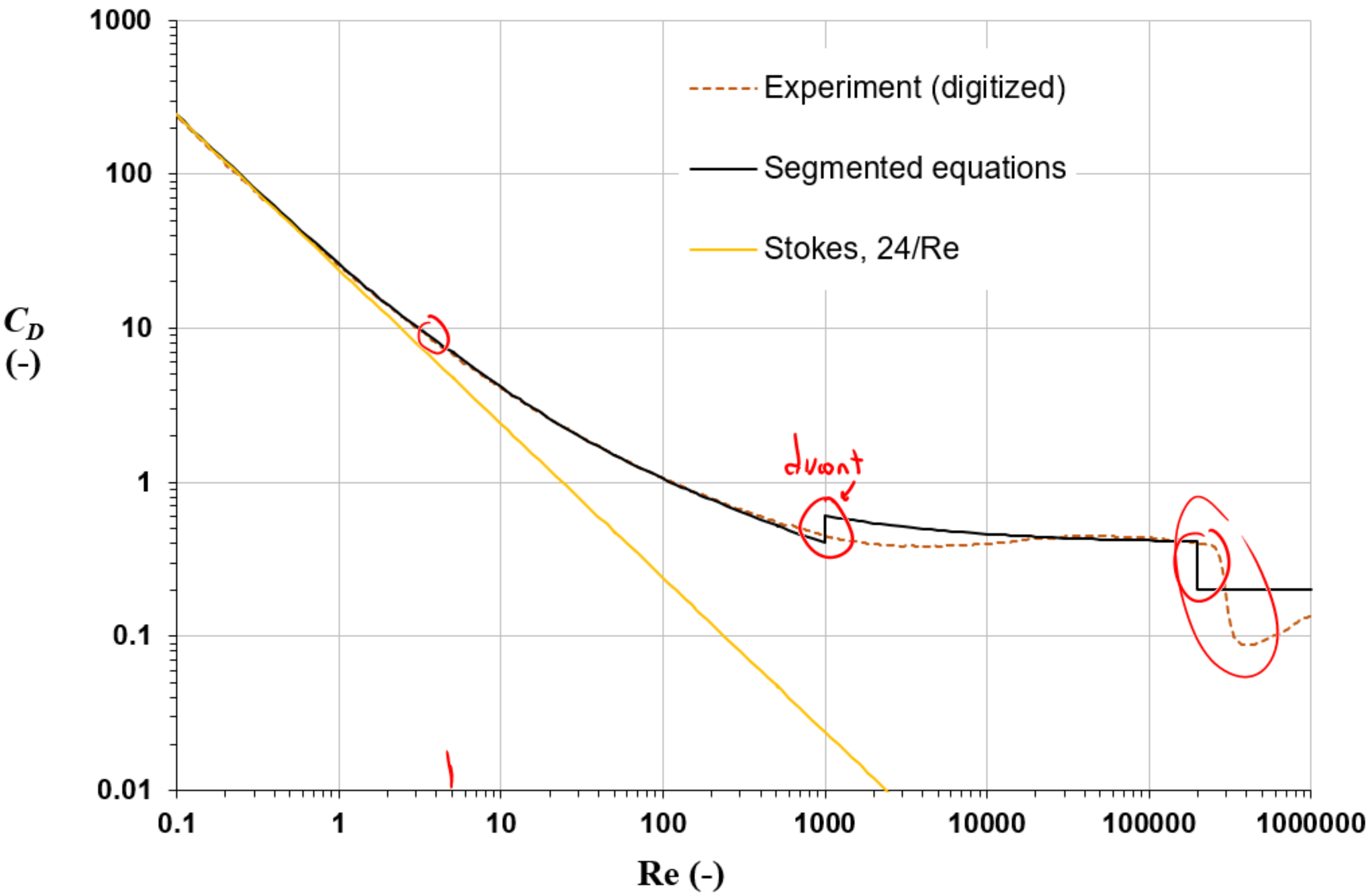
- $C_D = \dfrac{24}{\text{Re}}$ for Re < 0.1 where for all equations, $\text{Re} = \dfrac{\rho V D_p}{\mu} = \dfrac{V D_p}{\nu}$
- $C_D = \dfrac{24}{\text{Re}}\left(1 + 0.0916\,\text{Re}\right)$ for *any* Re < 5
- $C_D = \dfrac{24}{\text{Re}}\left(1 + 0.158\,\text{Re}^{2/3}\right)$ for 5 < Re < 1000
- $C_D = 0.4 + \dfrac{24}{\text{Re}} + \dfrac{6}{1+\sqrt{\text{Re}}}$ for 1000 < Re < 2×10^5
- $C_D \approx 0.2$ for Re > 2×10^6

Can combine segmented eq's. Use IF statements.

e.g., in Excel: =IF(B5<5, 24*(1+0.0916*B5)/B5, IF(B5<1000, 24*(1+0.158 ...)))

Nested IF

Problem: If use these in an IF statement, there are discontinuities.



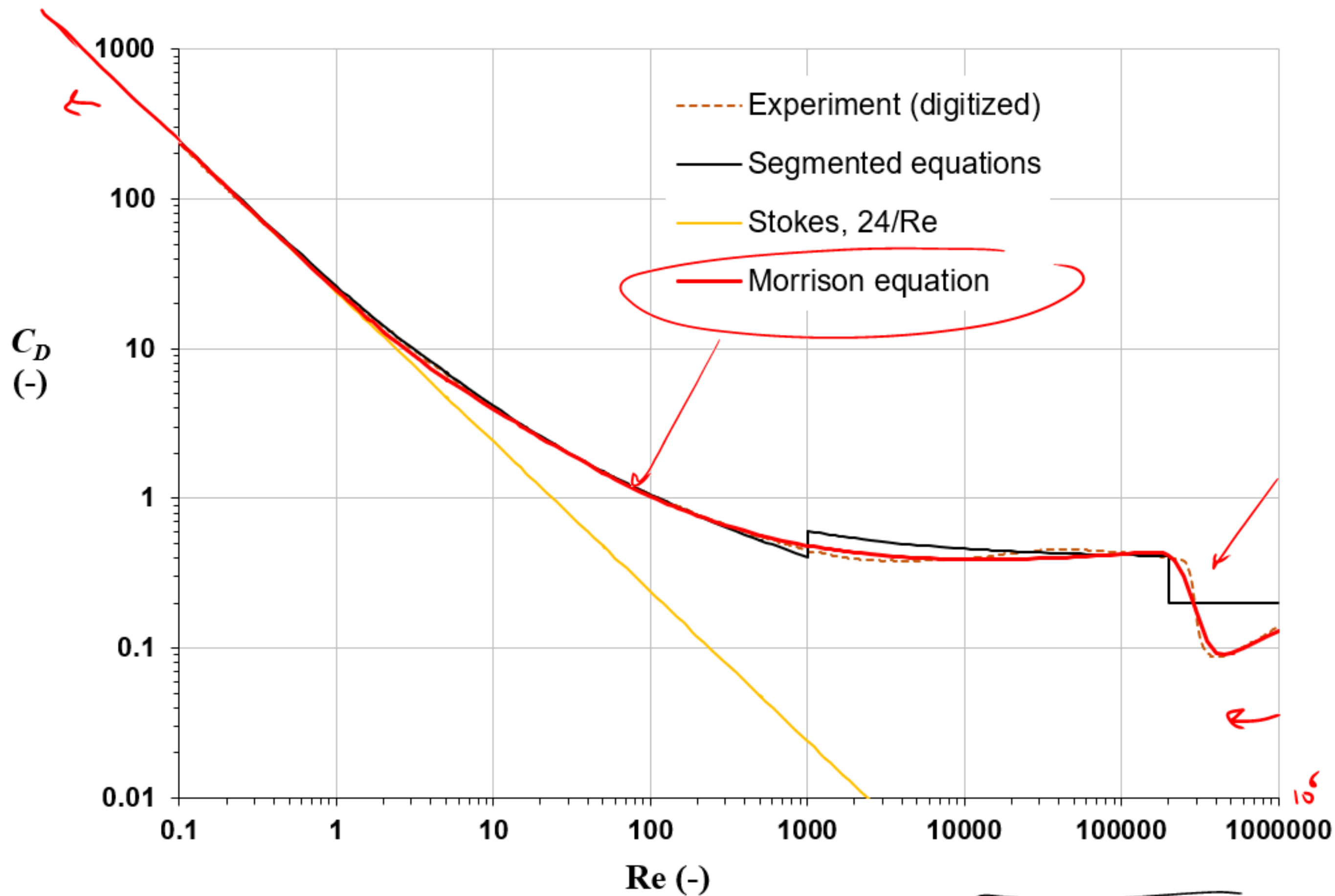
Fortunately, there is a 2016 paper by **Faith A. Morrison** where she created a curve fit equation that spans the entire range of Reynolds number up to 10^6. Here is the equation:

$$C_D \approx \frac{24}{\text{Re}} + \frac{2.6\left(\frac{\text{Re}}{5.0}\right)}{1+\left(\frac{\text{Re}}{5.0}\right)^{1.52}} + \frac{0.411\left(\frac{\text{Re}}{2.63\times10^5}\right)^{-7.94}}{1+\left(\frac{\text{Re}}{2.63\times10^5}\right)^{-8.00}} + \frac{0.25\left(\frac{\text{Re}}{10^6}\right)}{1+\left(\frac{\text{Re}}{10^6}\right)} \quad \text{for Re} < 10^6$$

Explicit Accurate

WE WILL ALWAYS USE THIS MORRISON EQ FOR CD

Here is a plot of $C_D(\text{Re})$ comparing Stokes, our segmented equations, and Morrison:



Compare Stokes & Morrison:

Re	C_D (Stokes)	C_D (Morrison)
0.01	2400	2400.15
0.1	240	240.22
1.0	24	24.673
10.0	2.4	3.9676

↓ never use Stokes

Example: Drag coefficient on a sphere

Given: A 1.55 mm sphere is moving in air at a speed of 1.25 m/s. The air properties are:

- $\rho = 1.246\ \text{kg/m}^3$
- $\nu = 1.426\times10^{-5}\ \text{m}^2/\text{s}$

$$\left[\nu = \frac{\mu}{\rho}\right]$$

To do: Calculate the Reynolds number and the drag coefficient for this sphere.

Solution:

using Morrison

$$\mathrm{Re} = \frac{VD_p}{\nu}$$

$$C_D \approx \frac{24}{\mathrm{Re}} + \frac{2.6\left(\frac{\mathrm{Re}}{5.0}\right)}{1+\left(\frac{\mathrm{Re}}{5.0}\right)^{1.52}} + \frac{0.411\left(\frac{\mathrm{Re}}{2.63\times10^5}\right)^{-7.94}}{1+\left(\frac{\mathrm{Re}}{2.63\times10^5}\right)^{-8.00}} + \frac{0.25\left(\frac{\mathrm{Re}}{10^6}\right)}{1+\left(\frac{\mathrm{Re}}{10^6}\right)} \quad \text{for } \mathrm{Re} < 10^6$$

unity conv. factor

$$\mathrm{Re} = \frac{(1.25\ \text{m/s})(1.55\ \text{mm})}{1.426\times10^{-5}\ \text{m}^2/\text{s}}\left(\frac{1\ \text{m}}{1000\ \text{mm}}\right) = 135.869\ \mathrm{Re}$$

$C_D =$ Put this in Excel, Matlab, ... software

$$C_D = 0.90149$$

IN EXCEL,

= 24/B5 + (2.6*(B5/5.0))/(1+((B5/5.0)^1.52)) + ~