Equation Sheet for M E 433 Quizzes and Exams

Author: John M. Cimbala, Penn State University. Latest revision, 05 January 2015

Note: This equation sheet will be provided in electronic form as a pop-up window during the quizzes taken at the Testing Center. ***Do not bring a printout of this equation sheet to the quiz***. This equation sheet has information for the entire semester ***in order of presentation***; for the earlier quizzes, ignore the later pages. This sheet is also useful for homework, in-class poll questions, and the open-book portion of the final exam.

General and conversions: $g = 9.807\ \frac{\text{m}}{\text{s}^2}$, $\frac{0.3048\ \text{m}}{1\ \text{ft}}$, $\frac{1\ \text{mile}}{1.6093\ \text{m}}$, $\frac{1\ \text{kPa}\cdot\text{m}^2}{1\ \text{kN}}$, $\frac{1\ \text{kN}\cdot\text{m}}{1\ \text{kJ}}$, $\frac{1\ \text{kW}\cdot\text{s}}{1\ \text{kJ}}$, $\frac{1\ \text{Btu}}{1.055056\ \text{kJ}}$, $\frac{1\ \text{kg}}{2.205\ \text{lbm}}$, $\frac{1\ \text{ton}}{2000\ \text{lbm}}$, $\frac{1\ \text{tonne (metric ton)}}{1000\ \text{kg}}$, $\frac{1\ \text{g}}{10^6\ \mu\text{g}}$, $\frac{1\ \text{m}}{10^6\ \mu\text{m}}$, $\frac{1\ \text{m}}{10^9\ \text{nm}}$, $V_{\text{sphere}} = \frac{4}{3}\pi\left(R_p\right)^3 = \frac{1}{6}\pi\left(D_p\right)^3$.

Molecular weights and mols: $m = nM$, $M_{\text{air}} = 28.97\ \text{g/mol}$, $M_{\text{water}} = 18.02\ \text{g/mol}$, Avagadro's number: 6.0225×10^{23}.

Volume and mass flow rate: $Q = \dot{V} = UA_c$, $\dot{m} = \rho Q = \rho\dot{V}$, $Q_{\text{STP}} = Q_{\text{actual}}\frac{P}{P_{\text{STP}}}\frac{T_{\text{STP}}}{T}$, $P_{\text{STP}} = 101.325\ \text{kPa} = 760\ \text{mm Hg}$, $T_{\text{STP}} = 25^\circ\text{C} = 298.15\ \text{K}$.

Ideal gas: $PV = nR_uT$, $R = R_u/M$, $PV = mRT$, $P = \rho RT$, $R_u = 8.314\frac{\text{kJ}}{\text{kmol}\cdot\text{K}}$, $R_{\text{air}} = 0.287\frac{\text{kJ}}{\text{kg}\cdot\text{K}}$.

Ideal gas mixture: $m_t = \sum_{j=1}^{J} m_j$, $n_t = \sum_{j=1}^{J} n_j$, $P = \sum_{j=1}^{J} P_j$, $V = \sum_{j=1}^{J} V_j$, $f_j = \frac{m_j}{m_t} = y_j\frac{M_j}{M_t}$, $y_j = \frac{n_j}{n_t} = \frac{P_j}{P} = \frac{V_j}{V}$, $M_j = \frac{m_j}{n_j}$, $PV = n_tR_uT$, $P_jV = n_jR_uT$, $PV_j = n_jR_uT$, $M_t = \sum_{j=1}^{J}\left(y_jM_j\right)$, $c_j = \frac{m_j}{V}$, $c_{\text{molar},j} = \frac{n_j}{V} = \frac{c_j}{M_j}$, $c_j = y_j\frac{M_j}{R_u}\frac{P}{T}$, $\dot{m}_j = c_jQ$.

Relative humidity and vapor pressure: $RH = \frac{P_{H_2O}}{P_{v,H_2O}}\times100\% = \frac{P_{H_2O}}{P_{\text{sat},H_2O}}\times100\%$, $y_{H_2O} = \frac{P_{H_2O}}{P_{\text{atm}}}$, $P_v = P_{\text{sat}}$ for any VOC.

Emission factors (EPA AP-42 and CHIEF): $EF = \frac{m_{\text{pollutant}}}{m_{\text{raw material or product produced}}}$, $\dot{m}_{\text{discharged}} = (1-E)\dot{m}_{\text{generated}}$, E = APCS removal efficiency.

Combustion of hydrocarbons: Stoichiometric equation, $C_xH_y + a_{\text{stoich}}\left(O_2 + 3.76N_2\right) \rightarrow bCO_2 + cH_2O + dN_2$,

Simple dry air = 21% O_2, 79% N_2, $\Phi = \frac{(F/O_2)_n}{(F/O_2)_{\text{n, stoichiometric}}} = \frac{a_{\text{stoich}}}{a}$, where a = molar coeffient of oxidizer O_2,

Flux chamber: $\frac{dm_j}{dt} = V\frac{dc_j}{dt} = c_{j,a}Q_a + S_j - c_jQ_a$, $\dot{m}_{j,\text{ generated}} = S_j = \left(c_{j,\text{ss}} - c_{j,a}\right)Q_a$, $EF = \frac{\dot{m}_{j,\text{ generated}}}{\text{some appropriate denom.}}$.

Tank filling: $\dot{m}_{j,\text{ displaced}} = \frac{M_jP_j}{R_uT}Q_{\text{liquid in}}$. **Coriolis force**: $\vec{F}_c = -2m\left(\vec{\Omega}\times\vec{U}\right)$.

Lapse rate: $\Gamma = -\frac{dT}{dz}$, Normal lapse rate $= 6.5\frac{^\circ\text{C}}{\text{km}}$, Dry adiabatic lapse rate $= 9.8\frac{^\circ\text{C}}{\text{km}}$.

Gradient diffusion: $J_A = -b\frac{da}{dz}$, where $a = \frac{A}{V}$ and A = mass, energy, momentum, … For mass, $M_jJ_j = -D_{aj}\frac{dc_j}{dz}$.

Gaussian plume model: $\sigma_y = ax^b$, $\sigma_z = cx^d + f$, with x in units of km and σ_y and σ_z in units of m.

- Absorbing ground: $c_j = \frac{\dot{m}_{j,s}}{2\pi U\sigma_y\sigma_z}\exp\left\{-\frac{1}{2}\left[\left(\frac{y}{\sigma_y}\right)^2 + \left(\frac{z-H}{\sigma_z}\right)^2\right]\right\}$, where $H = h_s + \delta h$ = effective stack height.
- Reflecting ground: $c_j = \frac{\dot{m}_{j,s}}{2\pi U\sigma_y\sigma_z}\left[\exp\left\{-\frac{1}{2}\left[\left(\frac{y}{\sigma_y}\right)^2 + \left(\frac{z-H}{\sigma_z}\right)^2\right]\right\} + \exp\left\{-\frac{1}{2}\left[\left(\frac{y}{\sigma_y}\right)^2 + \left(\frac{z+H}{\sigma_z}\right)^2\right]\right\}\right]$.

- Absorbing ground w/ inversion: $c_j = \frac{\dot{m}_{j,s}}{2\pi U \sigma_y \sigma_z} \exp\left[-\frac{1}{2}\left(\frac{y}{\sigma_y}\right)^2\right]\left\{\exp\left[-\frac{1}{2}\left(\frac{z-H}{\sigma_z}\right)^2\right] + \exp\left[-\frac{1}{2}\left(\frac{z-(2H_T - H)}{\sigma_z}\right)^2\right]\right\}$,

  where H = effective stack height and H_T is the elevation of the reflecting part of the inversion.

- Fumigating, reflecting ground, far downwind: $c_{j,F} = \frac{\dot{m}_{j,s}}{\sqrt{2\pi} U \sigma_y H_T} \exp\left[-\frac{1}{2}\left(\frac{y}{\sigma_y}\right)^2\right]$; at $y = 0$, $c_{j,F} = \frac{\dot{m}_{j,s}}{\sqrt{2\pi} U \sigma_y H_T}$.

Table 20.1 Stability Classifications*

Surface Wind Speed[a] m/s	Day Incoming Solar Radiation			Night Cloudiness[e]	
	Strong[b]	Moderate[c]	Slight[d]	Cloudy (≥4/8)	Clear (≤3/8)
<2	A	A–B[f]	B	E	F
2–3	A–B	B	C	E	F
3–5	B	B–C	C	D	E
5–6	C	C–D	D	D	D
>6	C	D	D	D	D

[a] Surface wind speed is measured at 10 m above the ground.
[b] Corresponds to clear summer day with sun higher than 60° above the horizon.
[c] Corresponds to a summer day with a few broken clouds, or a clear day with sun 35-60° above the horizon.
[d] Corresponds to a fall afternoon, or a cloudy summer day, or clear summer day with the sun 15–35°.
[e] Cloudiness is defined as the fraction of sky covered by clouds.
[f] For A–B, B–C, or C–D conditions, average the values obtained for each.

* A = Very unstable   D = Neutral
B = Moderately unstable   E = Slightly stable
C = Slightly unstable   F = Stable
Regardless of wind speed, Class D should be assumed for overcast conditions, day or night.

Table 20.2 Values of Curve-Fit Constants for Calculating Dispersion Coefficients as a Function of Downwind Distance and Atmospheric Stability

Stability	*a*	*b*	*x* < 1 km *c*	*x* < 1 km *d*	*x* < 1 km *f*	*x* > 1 km *c*	*x* > 1 km *d*	*x* > 1 km *f*
A	213	0.894	440.8	1.941	9.27	459.7	2.094	–9.6
B	156	0.894	106.6	1.149	3.3	108.2	1.098	2.0
C	104	0.894	61.0	0.911	0	61.0	0.911	0
D	68	0.894	33.2	0.725	–1.7	44.5	0.516	–13.0
E	50.5	0.894	22.8	0.678	–1.3	55.4	0.305	–34.0
F	34	0.894	14.35	0.740	–0.35	62.6	0.180	–48.6

Adapted from Martin, 1976.

Gaussian puff diffusion model: $\sigma_{xi} = \sigma_{yi} = ax^b$, $\sigma_{zi} = cx^d$, but x in units of m, not km, and σ_{yi} and σ_{zi} in units of m.

Use these empirical values for the instantaneous diffusion coefficients, depending on atmospheric stability conditions:

Stability condition	*a*	*b*	*c*	*d*
Unstable	0.14	0.92	0.53	0.73
Neutral	0.06	0.92	0.15	0.70
Very stable	0.02	0.89	0.05	0.61

Adapted from Slade (1968), as found in Heinsohn and Kabel (1999).

- Absorbing ground: $c_j(x,y,z,t) = \frac{m_j}{\pi\sqrt{2\pi}\sigma_{xi}\sigma_{yi}\sigma_{zi}} \exp\left\{-\frac{1}{2}\left[\left(\frac{x-Ut}{\sigma_{xi}}\right)^2 + \left(\frac{y}{\sigma_{yi}}\right)^2 + \left(\frac{z-H}{\sigma_{zi}}\right)^2\right]\right\}$.

- Ground level dose, absorbing ground: $D_j(x,y,0) = \frac{m_j}{\pi U \sigma_{yi}\sigma_{zi}} \exp\left\{-\frac{1}{2}\left[\left(\frac{y}{\sigma_{yi}}\right)^2 + \left(\frac{H}{\sigma_{zi}}\right)^2\right]\right\}$, (double for reflecting).

Particles: $c_{\text{number},j} = \dfrac{c_j}{m_{p,\text{mean}}}$, $m_{p,\text{mean}} = \rho_p \dfrac{1}{6}\pi\left(D_{p,am}(\text{mass})\right)^3$, $\vec{F}_{\text{gravity}} = (\rho_p - \rho)\dfrac{\pi}{6}D_p^3\vec{g}$, $\vec{F}_{\text{drag}} = -\dfrac{\rho}{8}\dfrac{C_D}{C}\pi D_p^2 \vec{v}_r |\vec{v}_r|$.

Relative particle velocity = $\vec{v}_r = \vec{v} - \vec{U}$, where $\vec{v}$ is the particle velocity and $\vec{U}$ is the air velocity.

Equation of particle motion: $\dfrac{d\vec{v}}{dt} = \dfrac{\rho_p - \rho}{\rho_p}\vec{g} - \dfrac{3}{4}\dfrac{\rho}{\rho_p}\dfrac{C_D}{C}\dfrac{1}{D_p}\vec{v}_r|\vec{v}_r|$, where C is the **Cunningham correction factor**,

$\text{Kn} = \dfrac{\lambda}{D_p}$, $\lambda = \dfrac{\mu}{0.499}\sqrt{\dfrac{\pi}{8\rho P}}$, $C = 1 + \text{Kn}\left[2.514 + 0.80\exp\left(-\dfrac{0.55}{\text{Kn}}\right)\right]$, and $C_D = C_D(\text{Re})$, where $\text{Re} = \dfrac{\rho|\vec{v}_r|D_p}{\mu}$,

$C_D = \dfrac{24}{\text{Re}}$ for Re < 0.1, $C_D = \dfrac{24}{\text{Re}}(1 + 0.0916\,\text{Re})$ for 0.1 < Re < 5, $C_D = \dfrac{24}{\text{Re}}\left(1 + 0.158\,\text{Re}^{2/3}\right)$ for 5 < Re < 1000.

Air at STP (25°C): $P_{\text{STP}} = 101.325\text{ kPa} = 760\text{ mm Hg}$, $\rho = 1.184\text{ kg/m}^3$, $\mu = 1.849\times10^{-5}\text{ kg/(m s)}$, $\lambda = 0.06704\ \mu\text{m}$.

Terminal settling velocity: $V_t = \sqrt{\dfrac{4}{3}\dfrac{\rho_p - \rho}{\rho}gD_p\dfrac{C}{C_D}}$, $\text{Re} = \dfrac{\rho V_t D_p}{\mu}$. For **Stokes flow** (Re<0.1), $V_t = \dfrac{\rho_p - \rho}{18}D_p^2 g\dfrac{C}{\mu}$.

Gravimetric settling in ducts: $L_c = \dfrac{HU}{V_t}$ = where V_t = function of D_p. **Grade efficiency**: $E(D_p) = 1 - \dfrac{c_j}{c_j(\text{in})}$:

Laminar settling: $E(D_p) = \dfrac{L}{L_c}$ if $L < L_c$; $E(D_p) = 1$ if $L > L_c$. Well-mixed settling: $E(D_p) = 1 - \exp\left(-\dfrac{L}{L_c}\right)$.

Gaussian Plume with Particle Settling: $c_j = \dfrac{\dot{m}_{j,s}}{2\pi U\sigma_y\sigma_z}\exp\left\{-\dfrac{1}{2}\left[\left(\dfrac{y}{\sigma_y}\right)^2 + \left(\dfrac{z - \left[H_0 - V_t\dfrac{x}{U}\right]}{\sigma_z}\right)^2\right]\right\}$, where $H_0 = H$ at $x = 0$.

Inertial Separation Devices:

Inertial "settling" velocity: $v_r = \sqrt{\dfrac{4}{3}\dfrac{\rho_p - \rho}{\rho}\dfrac{U_\theta^2}{r_m}D_p\dfrac{C}{C_D}}$, $\text{Re} = \dfrac{\rho v_r D_p}{\mu}$, where r_m = mean radius, $L_c = \dfrac{WU_\theta}{v_r}$, $\theta_c = \dfrac{L_c}{r_m}$,

Laminar settling: $E(D_p) = 1 - \dfrac{c_j}{c_j(\text{in})} = \dfrac{x}{L_c}$. Well-mixed settling: $E(D_p) = 1 - \dfrac{c_j}{c_j(\text{in})} = 1 - \exp\left(-\dfrac{x}{L_c}\right)$, where $x = r_m\theta$.

Standard Lapple cyclone: $D_{p,\text{cut}} = \sqrt{\dfrac{3\mu D_2^3}{128\pi Q(\rho_p - \rho)}}$, $E(D_p) = \dfrac{1}{1 + \left(\dfrac{D_p}{D_{p,\text{cut}}}\right)^{-2}}$, $\Delta P = 40.96\rho\left(\dfrac{Q}{WH}\right)^2$, $\dot{W}_{\text{blower}} = \dfrac{Q\Delta P}{\eta_{\text{blower}}}$.

Air Cleaners in Series and Parallel:

Parallel: $E(D_p)_{\text{overall}} = 1 - \sum_{j=1}^{m} f_j\left[1 - E(D_p)_j\right]$, where f_j = volume fraction through cleaner j, $f_j = \dfrac{Q_j}{Q_{\text{total}}}$.

Series: $E(D_p)_{\text{overall}} = 1 - \prod_{j=1}^{m}\left[1 - E(D_p)_j\right]$, where the volume flow rate of air through each cleaner is the same.

Rain, Spray Chambers, and Wet Scrubbers as Air Pollution Control Systems:

Single-drop collection grade efficiency: $E_d(D_p) = \left(\dfrac{r_1}{R_c}\right)^2 = \left(\dfrac{Stk}{Stk + 0.35}\right)^2$, where $Stk = \dfrac{(\rho_p - \rho)D_p^2(U_0 - V_{t,p})}{18\mu D_c}$.

Overall collection grade efficiency: $E(D_p) = 1 - \exp\left(-\dfrac{L}{L_c}\right)$, where $L_c = \dfrac{2}{3}\dfrac{Q_a}{Q_s}\dfrac{V_c}{V_{t,c}}\dfrac{D_c}{E_d(D_p)}$, $V_c = V_{t,c} - U_a$ for a ***spray chamber***. L_c must be estimated or calculated for a ***wet scrubber*** – depends on size and shape of the ***packing material***.

Air Filters: (ε = porosity, U_0 = air speed, L = filter thickness, $E_f(D_p)$ = single-fiber collection efficiency)

$$Stk = \frac{(\rho_p - \rho) D_p^{\,2} (U_0/\varepsilon)}{18\mu D_f}, \quad E_f(D_p) = \left(\frac{Stk}{Stk + 0.425}\right)^2, \quad L_c = \frac{\pi}{4}\frac{\varepsilon}{1-\varepsilon}\frac{D_f}{E_f(D_p)}, \quad E(D_p) = 1 - \exp\left(-\frac{L}{L_c}\right).$$

Electrostatic Precipitators: (ESPs)

$$E(D_p) = 1 - \exp\left(-\frac{L}{L_c}\right)$$, where L_c is dependent on voltages, particle composition, air speed, gap widths, and many other parameters. On quizzes and exams, L_c would either be given, or would be the variable to be calculate.

Polydisperse Aerosol Particle Statistics:

- j = class (bin) number with range $D_{p,\min,j} < D_p \le D_{p,\max,j}$, width $\Delta D_{p,j}$, and mid value $D_{p,j}$ for $j = 1$ to J.
- n_j = number of particles in bin j, and n_t = total number of particles in the sample, $n_t = \Sigma n_j$.
- $f(D_{p,j})$ = fraction of particles per bin width = $n_j/(\Delta D_{p,j}\, n_t)$.

Probability of particle in bin *j*: $\text{Prob} = f(D_{p,j}) \cdot \Delta D_{p,j} = \frac{n_j}{n_t}$, **Cumulative number distribution**: $F(a) = \int_0^a f(D_{p,j})\, dD_p$.

Median diameter: $F(D_{p,50}) = 0.50$. For number distribution, use $D_{p,50}$ (number); for mass use $D_{p,50}$ (mass).

Arithmetic mean diameter: $D_{p,am} = \int_0^\infty D_p f(D_p)\, dD_p = \frac{1}{n_t}\sum_{j=1}^{J}(n_j D_{p,j})$.

Geometric mean diameter: $D_{p,gm} = \left(D_{p,1}^{\,n_1} D_{p,2}^{\,n_2} \ldots D_{p,j}^{\,n_j} \ldots D_{p,J}^{\,n_J}\right)^{\frac{1}{n_t}} = \exp\left[\frac{1}{n_t}\sum_{j=1}^{J}\left(n_j \ln(D_{p,j})\right)\right]$, $D_{p,gm} = D_{p,50} = D_{p,\text{median}}$.

Geometric standard deviation: $\sigma_g = e^{\ln(\sigma_g)}$, $\ln(\sigma_g) = \sqrt{\frac{\sum_{j=1}^{J}\left\{n_j\left[\ln(D_{p,j}) - \ln(D_p)_{,am}\right]^2\right\}}{n_t - 1}}$, $\ln(D_p)_{,am} = \frac{\sum_{j=1}^{J}\left[n_j \ln(D_{p,j})\right]}{n_t}$.

Or, $\sigma_g = \frac{D_{p,50}}{D_{p,15.9}} = \frac{D_{p,84.1}}{D_{p,50}} = \sqrt{\frac{D_{p,84.1}}{D_{p,15.9}}}$, and σ_g is the same whether based on the number or the mass distribution. So, we can use *either* the number or mass values of $D_{p,50}$, $D_{p,15.9}$, and $D_{p,84.1}$, i.e., $\sigma_g = \frac{D_{p,50}(\text{number})}{D_{p,15.9}(\text{number})} = \frac{D_{p,50}(\text{mass})}{D_{p,15.9}(\text{mass})}$, etc., where $D_{p,gm}(\text{number}) = D_{p,50}(\text{number})$, and $D_{p,gm}(\text{mass}) = D_{p,50}(\text{mass})$

Conversion from number distribution to mass distribution: $m_j = n_j \rho_p \frac{\pi}{6}(D_{p,j})^3$. **Mass fraction**: $g(D_{p,j}) = \frac{m_j}{m_t}$, but we plot histograms as $\frac{g(D_{p,j})}{\Delta D_{p,j}}$ for nonequal bin widths. **Cumulative mass distribution**: $G(a) = \int_0^a g(D_{p,j})\, dD_p$.

Since $\ln(D_{p,50}\ (\text{mass})) = \ln(D_{p,50}\ (\text{number})) + 3\left[\ln(\sigma_g)\right]^2$, $\sigma_g = \exp\left\{\sqrt{\frac{\left[\ln(D_{p,50}\ (\text{mass})) - \ln(D_{p,50}\ (\text{number}))\right]}{3}}\right\}$.

Overall particle removal efficiency:

$$E_{\text{overall}} = \sum_{j=1}^{J}\left[E(D_{p,j})\frac{m_j}{m_t}\right] = \sum_{j=1}^{J}\left[E(D_{p,j})\frac{c_j}{c_{\text{overall}}}\right].$$