Due: In class, Friday **January 18, 2019**	**Name**	**PSU ID (abc1234)**

M E 522
Spring Semester, 2019
Homework Set # 1

Professor J. M. Cimbala

For instructor or TA use only:		
Problem	**Score**	**Points**
1		15
2		15
3		70
Total:		**100**

1. (15 pts) [Warm-up problem to get you back into shape after the break!] Consider the expression $\vec{\nabla}\times\left(\vec{\nabla}\phi\right)$ where scalar ϕ is a smooth continuous function of space.

(a) Re-write the expression in tensor notation.
(b) Staying in tensor notation, reduce the expression as far as possible.
(c) Expand out the expression in Cartesian coordinates (for consistency use i_1, i_2, i_3 instead of i, j, k and use x_1, x_2, x_3 instead of u, v, w). If you do it correctly, your answer should agree with that of Part (b).

2. (15 pts) Consider an infinitesimally small fluid element in the boundary layer along an **axisymmetric body**. Assume that boundary layer thickness δ is much smaller than r_0, where r_0 is the perpendicular distance from the axis of symmetry to the wall of the body (the wall is where we set $y = 0$ in boundary layer coordinates). By carefully summing the mass flow through each surface of your element (and making appropriate approximations as the element shrinks to zero size), show that the continuity equation reduces to $\boxed{\frac{\partial}{\partial x}\left(r_0 u\right)+r_0\frac{\partial v}{\partial y}=0}$.

3. (70 pts) Consider **Falkner-Skan** boundary layer flow in which the potential outer flow speed along the wall is ***U*(*x*)**. In other words, the outer flow speed is *not* constant as it was for the Blasius boundary layer. It turns out that a similarity solution can be found for this *general* boundary layer problem. Consider steady, incompressible, two-dimensional, laminar boundary layer flow over some semi-infinite body where the potential outer flow $U(x)$ is *known* and there is no length scale in the problem. Choose a similarity variable, $\boxed{\eta=\frac{y}{\delta_c(x)}}$, and let $\boxed{f'(\eta)=\frac{u(x,y)}{U(x)}}$, where δ_c is some fraction of the 99% boundary layer thickness δ. [I call it δ_c to indicate some *characteristic* boundary layer thickness, and to emphasize that it is *not* equal to the standard 99% boundary layer thickness.]

(a) Showing all your work, show that $\boxed{v=U\frac{d\delta_c}{dx}\eta f'-\frac{d\left(U\delta_c\right)}{dx}f}$. *Hint*: Use the chain rule, product rule, and reverse product rule as necessary to get your answer in the form above. Also apply the boundary condition $f(0) = 0$ for consistency (this boundary condition is arbitrary, but since f is proportional to the stream function, it makes sense to set it to zero at the wall).
(b) Plug the above expression for v into the x-momentum boundary layer equation to generate the final similarity equation, again showing all your work, and putting it in standard ODE form. The final result should be $\boxed{f'''+\alpha ff''+\beta[1-(f')^2]=0}$ where α and β are in general functions of x, U, kinematic viscosity ν, and δ_c. But, as is typical for similarity problems, we force α and β to be *constants* so that the final similarity equation is a function of η only. Write the equations for α and β as functions of x, U, ν, and δ_c. *Caution*: In Microsoft Word, it is difficult to distinguish between Times New Roman lower case italic "vee," v, and Greek letter "nu," ν.

Note: There is another page. →

(**c**) In ME 521, we generated and solved the famous Blasius flat plate boundary layer similarity solution in which $U(x) = U$ = constant. The Blasius ODE reduces to $\boxed{f''' + cff'' = 0}$, where c is an arbitrary constant. Most authors set c to either 1 or ½. For now, just leave it as constant c. It turns out that the Blasius solution is a special (simpler) case of Falkner-Skan! For what values of α and β does the Falkner-Skan equation reduce to the Blasius equation? Verify your value of β by applying your equation for β from Part (b). Then, using your equation for α from Part (b), integrate to generate an expression for $\cancel{\alpha}$ [Correction – I meant δ_c not α!] as a function of c, ν, x, and U.

(**d**) Now consider another special case of Falkner-Skan in which $U(x)$ and $\delta_c(x)$ are **power law functions** of streamwise boundary layer coordinate x. Specifically, set $\boxed{U(x) = Bx^m}$ and $\boxed{\delta_c(x) = Cx^b}$, where B, C, b, and m are all constants. Similarity solutions can be found for this power law case, but only for certain restricted cases. What conditions must exponents m and b satisfy in order for similarity solutions to be possible? (In other words, find a *necessary* relationship between m and b such that similarity solutions are possible.)

(**e**) For the case in which we choose $\alpha = 1$ and we let β remain as an arbitrary constant (we call β the ***Falkner-Skan parameter***), the similarity equation of Part (b) becomes $\boxed{f''' + ff'' + \beta[1-(f')^2] = 0}$, which is the standard ***Falkner-Skan similarity equation***. Write the appropriate boundary conditions for this case.

(**f**) One application of the Falkner-Skan similarity solution is boundary layer flow over an infinite 2-D wedge of angle γ as sketched. From potential flow theory, the potential outer flow for this case is $\boxed{U(x) = \text{constant} \cdot x^{n-1}}$, where n is related to wedge angle γ by $\boxed{\gamma = 2\pi\left(\frac{n-1}{n}\right)}$. Generate an expression for exponent m as a function of γ in the power law function of Part (d). When $\gamma = 90^o$, what is the value of m?