Due: In class, Friday **February 8, 2019**	**Name**	**PSU ID (abc1234)**

ME 522
Spring Semester, 2019
Homework Set # 3

Professor J. M. Cimbala

For instructor or TA use only:		
Problem	**Score**	**Points**
1		10
2		40
3		20
4		30
Total:		**100**

1. (10 pts) *Note*: This is a quick warm-up problem to practice for Exam 1. Consider *spatial* instability, where disturbances can grow or decay in the x-direction, while remaining periodic in time. Consider a linear stability analysis with normal mode disturbances of the form $v(x,y,t)=\hat{v}(y)e^{ik(x-ct)}$. Wavenumber k may be complex, i.e., $k = k_r + ik_i$. For ***spatial instability analysis***, circle the correct choices:

(a) T F Parameter c may be complex. [Circle T (true) or F (false)]

(b) The product of c and k must be (real complex imaginary).

(c) If k_i is (positive negative zero), the basic state is **stable**.

(d) If k_i is (positive negative zero), the basic state is **unstable**.

(e) If k_i is (positive negative zero), the basic state is **neutrally stable**.

2. (40 pts) Consider the example secondary flow problem discussed in class, namely a semi-infinite flat plate in the x-z plane with the outer flow given by $U(x) = U$ = constant and $W(x) = U\cdot(a - bx)$, where a and b are constants. In class it was shown that the continuity and x-momentum equations are identical to those of the simple 2-D Blasius flat plate boundary layer. A similarity solution (the famous Blasius solution) is thus possible for continuity and x-momentum since they are uncoupled from z-momentum.

(a) Show that the z-momentum equation reduces to $u\dfrac{\partial w}{\partial x}+v\dfrac{\partial w}{\partial y}=-U^2b+\nu\dfrac{\partial^2 w}{\partial y^2}$.

(b) Let $w=W(x)f'(\eta)-Ubxh(\eta)$, where $h(\eta)$ is some new dependent similarity variable. Show that the z-momentum equation reduces to $h''+\dfrac{h'f}{2}-hf'+1-(f')^2=0$. *Hint*: At one point you will need to plug in the Blasius equation to cancel out two terms – kind of a cool trick.

(c) Write out the appropriate boundary conditions for $h(\eta)$, and explain how this problem can be solved with the Runge-Kutta technique. Specifically, set up the R-K equations to solve both the Blasius equation and the z-momentum similarity equation *simultaneously*. Be sure to indicate which boundary conditions can be specified, and which must be "guessed."

(d) Solve this problem on the computer. For full credit, turn in a printout of your code, the values of your guessed BCs that yield the correct solution, and a plot of (at minimum) $f'(\eta)$ and $h(\eta)$. You may use Professor Cimbala's Blasius Matlab program, available on the website, as a starting point.

(e) Verify that the secondary flow is in the expected direction (look at the sign of velocity component w).

3. (20 pts) Do a little library or Internet searching and find another example of secondary flow in three-dimensional boundary layers (an example not specifically discussed in class). Explain the direction of the secondary flow and some physics of how it sets itself up. I am not looking for a long, detailed explanation or solution – just some sketches or pictures (perhaps even links to videos) and a brief explanation. If you have a secondary flow problem and solution that could be made into a homework problem in a future semester of ME 522, attach a copy of the solution since it may be helpful in the future. Examples that are related to your research project are particularly desired.

Note: There is another page. →

4. (30 pts) Linearized stability analysis is also useful for *computational* fluid dynamics. To illustrate, consider a simple differential equation for $\tilde{u} = \tilde{u}(x,t)$, namely, $\frac{\partial \tilde{u}}{\partial t} + b\frac{\partial \tilde{u}}{\partial x} = 0$, where b is a constant. Suppose we are solving this equation numerically with a structured mesh, using constant mesh size in x $(x = j\Delta x)$, and constant time step Δt $(t = n\Delta t)$, where j and n are integers representing the node number in x and the number of time steps, respectively.

(**a**) Write the above differential equation in finite difference form, using first-order forward differences in time, and second-order central differences in space. For consistency of notation, take all derivatives around grid point j and time step n, and denote these as $\left.\frac{\partial \tilde{u}}{\partial t}\right)_j^n$ and $\left.\frac{\partial \tilde{u}}{\partial x}\right)_j^n$, where the subscript denotes grid point j, and the superscript denotes time step n.

(**b**) Just as in a physical problem, numerical solutions suffer from "random noise," namely truncation errors, etc., which can grow or decay in time. Suppose some very small errors, $u(x,t)$ are introduced into the solution. Obtain the linearized disturbance equation for disturbances $u(x,t)$. Again for consistency, use the same notation that we used in class, namely, $\tilde{u}(x,t) = U(x,t) + u(x,t)$, where U is the basic state and $u(x,t)$ is the disturbance. Write the linearized disturbance equation in finite difference form.

(**c**) Although numerical noise is "random," let's assume the noise is of the form $u(x,t) = \hat{u}(t)\cdot e^{i(kx-\omega t)}$, where k is the wave number and ω is the radian frequency. (In other words, we can employ the method of normal modes here, with the idea that any actual disturbance can be constructed by superposition.) Also define the ratio of numerical error between any two successive time steps as $g = u_j^{n+1}/u_j^n$. Generate an expression for g. **Hint**: Your answer should contain two terms, one of which contains $\sin(k\Delta x)$.

(**d**) Analyze the temporal stability of this problem. **Hint**: Look at the magnitude of g. Will disturbances (truncation errors, etc.) grow or decay in time? At which wave number(s) will disturbances grow the fastest?