

Due: In class, Friday March 15, 2019	<u>Name</u>	<u>PSU ID (abc1234)</u>
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ME 522
Spring Semester, 2019
Homework Set # 6

Professor J. M. Cimbala

For instructor or TA use only:		
Problem	Score	Points
1		15
2		20
3		20
4		45
Total:		100

- (15 pts) For any two total turbulence variables $\tilde{a} = A + a$ and $\tilde{b} = B + b$, the **correlation** of the fluctuations is defined as \overline{ab} . The **correlation coefficient** is a nondimensional form of the correlation, defined as $C_{ab} = \frac{\overline{ab}}{\sqrt{\overline{a^2} \overline{b^2}}}$, where $\overline{a^2}$ and $\overline{b^2}$ are the **autocorrelations** of the fluctuations. In a study of atmospheric turbulence, the root mean square (rms) of the vertical velocity fluctuations is measured: $\sqrt{\overline{w^2}} = 1.4$ m/s, and the rms of the temperature fluctuations is measured: $\sqrt{\overline{T'^2}} = 0.16$ °C. The correlation coefficient between these two fluctuating variables is measured to be 0.68. The air is at 0°C.
 - Calculate the turbulent heat flux $\rho C_p \overline{wT'}$ in the vertical direction in units of W/m². Show all unit conversions and give your answer to two significant digits, which is about as much as we can ever expect for these kinds of estimates.
 - If the atmospheric boundary layer is 150 m high, and the temperature difference between the ground and the air above the boundary layer is 10°C, estimate the order of magnitude of the vertical component of the **laminar** heat flux $\bar{q} = -k\bar{\nabla}T$. Compare to the result of Part (a). Which heat flux is dominant, laminar or turbulent? Discuss why.
- (20 pts) Use the tabulated results of HW 4, Problem 3 (the Bénard convection CFD problem, even mode) for this problem. The table is posted on the course website as an Excel file. *Note:* The data include results from previous semesters as well, arranged in order of increasing L/d , regardless of semester. I deleted only the clear outliers in the students' results, so there may still be some erroneous "outliers". [These are real data, not "fake news."]
 - Plot nondimensional wavelength (λ/d) as a function of aspect ratio (L/d). Discuss. Is there an observable trend in the data? *Note:* For best results, plot symbols only – do not connect the data with line segments or some kind of smooth curve, which would make the plot much more confusing.
 - To help you understand what is going on, also plot L/λ vs. L/d , again as symbols only. Discuss. Specifically, provide a physical explanation for the shape of the plots.
- (20 pts) Consider turbulent flow using Reynolds decomposition: $\tilde{u}_i = U_i + u_i$, $\tilde{v}_i = V_i + v_i$, etc.
 - Show that $\overline{\tilde{u}\tilde{v}} = UV + \overline{uv}$. Write this equation in a handy place for future reference. [It will come in handy many times in the future.]
 - True or False? $\tilde{u}_{RMS} = \sqrt{\overline{\tilde{u}^2} + \tilde{u}_{SD}^2}$. Prove it.
 - True or False? $\tilde{u}_{RMS} = \overline{\tilde{u}} + \tilde{u}_{SD}$. Prove it.
 - Make up some random variables \tilde{u} and \tilde{v} with 10 values each. With these numbers, verify your answers to parts (a), (b), and (c) above. *Note:* Each student will have a different set of numbers, but the equations should work (if they are true) or not work (if they are false) regardless of the values you choose.
- (45 pts) Consider inviscid temporal stability analysis of a laminar far wake, as discussed in class. We manipulated the Rayleigh equation into a form that is **universal** for any wake defect so that we need to solve only once to find eigenvalue c_u for each k . In nondimensional form, the modified **universal wake Rayleigh equation for temporal instability** was given as

$$(W - c_u)(\phi_{yy} - k^2\phi) - W_{yy}\phi = 0$$

where

- $W(y)$ is the **normalized wake defect** $W(y) = \frac{U - U_\infty}{U - U_c}$
- w_c is the **nondimensional velocity defect at the centerline**
- k is the **nondimensional wavenumber**, and for temporal stability analysis, k is **real**
- c_u is the **nondimensional universal wave speed** $c_u = \frac{1}{w_c}(1 - c)$ (c_u may be complex, $c_u = c_{u_r} + ic_{u_i}$).

Note: There is another page. →

Experiments reveal that $W(y)$ is closely approximated by $W(y) = \exp(-ay^2)$, where $a = \ln(2)$ and we note that y and all the variables here are nondimensional. To solve this eigenvalue problem, we split up our Rayleigh equation into real and imaginary components, and then set up four dependent variables $Y_1, Y_2, Y_3,$ and Y_4 to solve using Runge-Kutta (R-K). It was noted in class that derivatives of the complex amplitude function ϕ appear in these R-K variables after splitting it into real and imaginary components, namely, $\phi_{r,yy}$ and $\phi_{i,yy}$.

- (a) Generate expressions for $\phi_{r,yy}$ and $\phi_{i,yy}$ in terms of ϕ_r, ϕ_i, W and its derivatives as needed, $k, c_{u,r}$, and $c_{u,i}$. *Hint:* It is most convenient to first write our modified Rayleigh equation in standard form for differential equations.
- (b) Solve using R-K for *your* value of k (see table for your assigned k). For consistency, march in the *negative* y direction from infinity (choose $y = 5$ as “infinity”) to 0, being careful of the BCs at the edge as discussed in class. You may use Matlab or any other software you wish, and you can solve either the split Rayleigh equation using all real variables as outlined in class, or you can modify the equations and solve using complex variables – your choice. Make sure you fill in the table at the appropriate place so that David can keep track of all the individual results, which we will plot later. *Note:* You will need to iterate, either manually or with some kind of automatic iteration technique if you are clever enough to do that [Matlab’s *fsolve* function is useful here]. If you need help with initial guesses for your case, ask either David or Professor Cimbala for a hint at something reasonably close. *Hint:* As a starting point for your initial guesses, $c_{u,r}$ is close to k for small k , and is about half of k for large k . $c_{u,i}$ starts at about -0.04 for small k , decreases to a min around -0.3 for the middle values of k , and then rises back to around -0.04 again when k is large.

PSU Login	Assigned k	$c_{u,r}$	$c_{u,i}$
sjal98@psu.edu	0.01		
agc86@psu.edu	0.02		
src347@psu.edu	0.03		
exc5257@psu.edu	0.04		
smc303@psu.edu	0.05		
tmc5478@psu.edu	0.06		
mrc453@psu.edu	0.07		
mje32@psu.edu	0.08		
kmh774@psu.edu	0.09		
iph3@psu.edu	0.1		
auh1002@psu.edu	0.12		
xuh128@psu.edu	0.14		
nuh98@psu.edu	0.16		
nxj190@psu.edu	0.2		
rtj10@psu.edu	0.25		
ajr545@psu.edu	0.3		
acs5213@psu.edu	0.35		
dxs5610@psu.edu	0.4		
cms685@psu.edu	0.45		
yzs220@psu.edu	0.5		
jxs2469@psu.edu	0.6		
emv53@psu.edu	0.7		
auw317@psu.edu	0.8		
ctw39@psu.edu	0.9		
djw68@psu.edu	1.0		
ajw324@psu.edu	1.1		
phw13@psu.edu	1.2		
hzx25@psu.edu	1.3		
jry40@psu.edu	1.4		

