Due: In class, Friday **March 22, 2019**	**Name**	**PSU ID (abc1234)**

ME 522
Spring Semester, 2019
Homework Set # 7

Professor J. M. Cimbala

For instructor or TA use only:		
Problem	**Score**	**Points**
1		20
2		20
3		60
Total:		**100**

1. (20 pts) In class we used the standard Boussinesq approximation of the momentum equation (nearly incompressible flow with buoyancy) and derived the following Reynolds-averaged momentum equation for the mean flow,

$$\boxed{\rho_0\left[\frac{\partial U_i}{\partial t}+U_j\frac{\partial U_i}{\partial x_j}\right]=-\frac{\partial P}{\partial x_i}-\rho_0 g\delta_{i3}\left[1-\alpha\left(\overline{T}-T_0\right)\right]+\frac{\partial}{\partial x_j}\left[\mu\frac{\partial U_i}{\partial x_j}-\rho_0\overline{u_i u_j}\right]}$$

I was looking at a book and found this form of the mean momentum equation for the same Boussinesq approximation,

$$\boxed{\frac{\partial U_i}{\partial t}+\frac{\partial}{\partial x_j}\left(U_i U_j\right)=\frac{1}{\rho_0}\frac{\partial\overline{\tau_{ij}}}{\partial x_j}-\frac{g}{\rho_0}\overline{\rho}\delta_{i3}}$$

where the author defines the mean stress tensor as

$$\boxed{\overline{\tau_{ij}}=-P\delta_{ij}+2\mu S_{ij}-\rho_0\overline{u_i u_j}}$$

and S_{ij} is the same strain rate tensor that we used in ME 521 and in Kundu et al's textbook. Are these two equations equivalent? Show why or why not.

2. (20 pts) Use the tabulated results of HW 6, Problem 4 (the Rayleigh wake instability problem) for this problem. The table will be posted on the course website as an Excel file as soon as it is available (hopefully by Monday).

(**a**) Plot the real and imaginary parts of universal wave speed (c_{u_r} and c_{u_i}) as functions of k. There may be some scatter and outliers since each data point was generated by a different student using a different code. Note that I also included data from a previous semester so that you have more data to work with (and potentially more scatter in the data).

(**b**) Discuss whether these results are consistent with Rayleigh's inflection point theorem and with Fjortoft's theorem for inviscid flow instability. Why or why not?

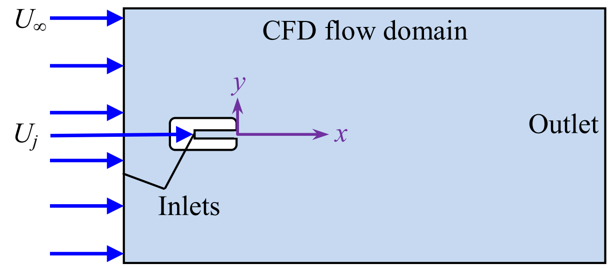
(**c**) In order to actually *use* these results and compare to experiment (as Professor Cimbala had to do for his PhD thesis research), we need to convert back to the *actual* (not the universal) wave speed c and wave number k. But we will stay nondimensional, so use $c_i=-w_c c_{u_i}$, $w_c=1-U_c/U_\infty$, and $k=k_d b$ where b is the characteristic width of the wake (b grows with x downstream). Plot dimensionless stability parameter c_i vs $k_d d$ for two cases on the same plot:

- wake defect w_c = 0.80 and nondimensional width b/d = 1.1 (near wake)
- wake defect w_c = 0.20 and nondimensional width b/d = 3.2 (far wake).

Compare the two curves and discuss how they agree (qualitatively) with experimental observations as shown in class; specifically the wavelengths, frequencies, and amplitudes of the amplified disturbances.

Note: There is another page. →

3. (60 pts) Let's have some fun with CFD! Compare the decay of turbulent kinetic energy in a ***weak wake***, a ***weak jet***, and a nearly ***momentumless wake***. (All cases are 2-D.) This is a *very* open-ended homework problem. Create a computational domain somewhat like the one sketched here (not to scale), but you can use any body shape you want (cylinder, airfoil, rectangle,...), any dimensions you want, any fluid you want, any code you want, any turbulence model you want (the default for most commercial codes is K-ε, which is fine), any velocities you want... Just be sure that the Reynolds number is high enough to ensure that the flow is turbulent and that the flow domain is long enough for the wakes to develop fully. I highly recommend that you employ symmetry about the x-axis (do the top half only since all three wakes are symmetric) to save computer time and to avoid potential convergence problems. [If you do the full flow (top and bottom), the CFD code may try to form Karman vortices and could encounter convergence issues.] *Note*: Based on my experience, it is best to have a section of channel inside the body with a velocity inlet on the left of the channel (as sketched) rather than trying to inject the jet fluid right at the trailing edge of the body. This allows for the channel flow to develop somewhat before exiting into the wake and leads to better stability of your solution. Run three cases with the same value of freestream velocity but three different jet velocities: (1) a weak wake (U_j/U_∞ not large enough to cancel the drag), (2) a weak jet (U_j/U_∞ larger than enough to cancel the drag), and (3) a nearly momentumless wake (U_j/U_∞ adjusted to nearly completely cancel the drag). In Case (3), the flow far downstream should have a mean velocity profile that looks approximately like that discussed in class for the momentumless wake case. To find the momentumless case, you can either adjust the jet speed by trial and error or you can more formally integrate the downstream velocity profile to calculate the net momentum deficit of the wake. To turn in for this homework:



(**a**) A sketch or computer drawing of your geometry and a list of dimensions, velocities, fluid properties, etc.

(**b**) Some plots of your results for all three cases for direct comparison. These may include, for example, velocity vector plots, pressure and/or velocity contour plots, and velocity profiles (U vs. y) at the same locations for the three cases.

(**c**) A contour plot of turbulent kinetic energy (K or tke) for all three cases. Do your results agree (at least qualitatively) with the class discussion about momentumless wakes, and in particular the ***dissipative nature of turbulence***? Discuss briefly.