Due: In class, Friday **April 5, 2019**	**Name**	**PSU ID (abc1234)**

ME 522
Spring Semester, 2019
Homework Set # 8

Professor J. M. Cimbala

For instructor or TA use only:		
Problem	**Score**	**Points**
1		15
2		15
3		10
4		20
5		30
6		10
Total:		**100**

NOTE: Some changes **in red** were made on 3/28 for clarity based on some student questions.

1. (15 pts) For fun, let's do a turbulent order of magnitude (o.o.m.) analysis of Jupiter's Giant Red Spot (GRS). Use the following estimates: diameter = 4×10^7 m, depth through atmosphere = 150 km, velocity scale of the large turbulent eddies = 50 m/s, length scale of the large turbulent eddies = 2×10^6 m. Assume the atmosphere is mostly hydrogen with average density = 30 kg/m^3 and average kinematic viscosity = 2×10^{-4} m^2/s.

(**a**) Estimate the kinetic dissipation rate per unit mass and all three of the Kolmogorov microscales (length, velocity, and time).

(**b**) Estimate the total dissipation rate in GW (gigawatts) and compare to the total installed power capacity of the largest hydroelectric dam in the world. Specifically, estimate how many of these hydro plants would be required to supply enough power to run the GRS.

2. (15 pts) Refer to the class notes on Reynolds decomposition. There we outlined a step-by-step procedure for finding both the mean equations of motion and the equations for the turbulent fluctuations. We discussed this for the continuity equation and the momentum equation. In similar fashion, starting with Equation (3) of the typed notes [Lecture 25, page 3] and showing all your algebra, generate both the mean energy equation (3m) and the energy equation for the turbulent fluctuations (3f). Put Eq. (3f) in a safe place also, for future reference.

3. (10 pts) The turbulent viscous dissipation rate ε (rate of energy loss per unit mass) is known to be a function of length scale ℓ and velocity scale u' of the large-scale turbulent eddies. Using dimensional analysis (Buckingham Pi and the method of repeating variables), and showing all your work, generate an expression for ε as a function of ℓ and u'. Comment on the power and usefulness of dimensional analysis.

4. (20 pts) The kinematic Reynolds stress transport equation was given in the class notes. The tke equation (tke budget) was also given. By definition, however, tke is one half of the sum of the diagonal components of the kinematic Reynolds stress tensor. So, theoretically, the tke equation should be obtainable by ***contraction*** of the kinematic Reynolds stress tensor. [Recall, contraction of tensor A_{ij} is accomplished by setting the two indices equal, i.e., the contraction of A_{ij} is A_{ii}, which is equivalent to summing the diagonals.] Contract the kinematic Reynolds stress equation to see if you can derive the tke equation. (Assume incompressible flow and ignore gravity for simplicity.) Note that you will need to "manipulate" the viscous terms to show that they are indeed equivalent.

Note: There is another page. →

5. (30 pts) Prepare a "mini lecture" about correlations and spectra as applied to turbulent flows. In particular, define and describe the ***correlation function***, the ***autocorrelation function***, the **integral time scale of turbulence**, and describe the **turbulent energy spectrum** and its significance. Most of this material is found in Section 12.4 of Kundu et al, Ed. 6. [The section is called ***Correlations and Spectra***, but you are welcome to use other books or online resources if you did not purchase the book.] In other words, pretend you are a professor and you need to teach this material. To teach effectively, you must first learn and understand the material yourself! List any references you used. Don't be concerned about how long your lecture is; rather, prepare enough so that *you* understand this material, and know how to explain the material in a way that is easy for your "students" to understand. Sketches are always good teaching tools. The basic idea of this homework problem is to provide incentive for you to go through this material in detail, since we do not have time to cover it in class. Also, many of you will end up being faculty members after graduation, and this gives you some experience in preparing a (high level) lecture. Maybe it will even help you decide whether you want to pursue an academic career or not!

6. (10 pts) Here is a fun problem. Take a picture of a turbulent flow. [It can be anything – something you observe as you go about your business, something related to a hobby of yours, something related to your research, etc.] Send the file to both David and Professor Cimbala, and also print it out for your homework submission. Briefly describe the flow (what it is, where you took the photo, etc.). Do your best to crudely estimate some kind of Reynolds number and the size of the smallest possible eddies in the flow.