Due: In class, Friday **April 12, 2019**	**Name**	**PSU ID (abc1234)**

ME 522
Spring Semester, 2019
Homework Set # 9

Professor J. M. Cimbala

For instructor or TA use only:		
Problem	**Score**	**Points**
1		15
2		10
3		15
4		30
5		20
6		10
Total:		**100**

1. (15 pts) Consider 2-D turbulent channel flow, as discussed in class. It was shown that the mean x-momentum equation reduces to an **o.d.e.**, $\boxed{\mu\frac{dU}{dy}-\rho\overline{uv}=-\tau_w\frac{y}{b}}$, where τ_w is a constant. At this point, some kind of turbulence model must be used to continue further. Suppose the K-ε turbulence model is used to solve this problem. A near-wall formulation of the t.k.e. equation is $\boxed{\frac{DK}{Dt}\equiv\frac{\partial K}{\partial t}+U_j\frac{\partial K}{\partial x_j}=\frac{\partial}{\partial x_j}\left[\left(\nu+\frac{\nu_e}{\sigma_k}\right)\frac{\partial K}{\partial x_j}\right]+\nu_e\left(\frac{\partial U_i}{\partial x_j}+\frac{\partial U_j}{\partial x_i}\right)\frac{\partial U_i}{\partial x_j}-\varepsilon}$, where $\boxed{\nu_e=C_\mu f_\mu\frac{K^2}{\varepsilon}}$, and f_μ is a ***wall damping function***, various forms of which exist in the literature.

(**a**) For this problem, reduce the t.k.e. equation as far as possible. Your final result should be an o.d.e. in the y-direction in terms of variables U, K, and ε, kinematic viscosity ν, kinematic eddy viscosity ν_e, and any necessary model constants.

(**b**) Express the Reynolds stress term, $-\rho\overline{uv}$, in the momentum equation as a function of y, U, K, ε, ρ, f_μ, and any necessary model constants.

2. (10 pts) In thin turbulent shear flows and boundary layer type flows which are stationary and 2-D in the mean (BLs, jets, wakes, mixing layers, etc.), the dominant mean shear is $\partial U/\partial y$, and the dominant Reynolds stress component is $-\rho_0\overline{uv}$. Consider two cases, the first with U increasing with y, and the second with U decreasing with y.

(**a**) For both cases, using physical arguments, predict the typical signs of $\overline{uv}$ and $\overline{uv}\frac{\partial U}{\partial y}$.

(**b**) Consider the term labeled Roman numeral V in the equations for mean kinetic energy and turbulent kinetic energy (t.k.e.), as provided in the class notes (Eqs. 1 and 3 of Lecture 26). What is the typical sign of this term for the mean kinetic energy equation? For the t.k.e. equation? For both equations, indicate whether this term contributes to a *gain* or a *loss* of kinetic energy. Does kinetic energy typically flow from the mean to the turbulence, or vice-versa? Discuss.

3. (15 pts) **Tennekes & Lumley's classic problem 1.4** (duplicated here). For consistency, do the following:

(**a**) Estimate the range of expected frequencies encountered by the hot wire (lowest and highest expected).

(**b**) Estimate the maximum allowable length of the hot wire.

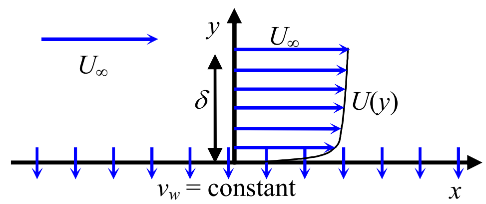
(**c**) Estimate the minimum acceptable noise level of the electronics as an equivalent turbulence intensity, assuming at least one order of magnitude between signal and noise.

> **1.4** An airplane with a hot-wire anemometer mounted on its wing tip is to fly through the turbulent boundary layer of the atmosphere at a speed of 50 m/sec. The velocity fluctuations in the atmosphere are of order 0.5 m/sec, the length scale of the large eddies is about 100 m. The hot-wire anemometer is to be designed so that it will register the motion of the smallest eddies. What is the highest frequency the anemometer will encounter? What should the length of the hot-wire sensor be? If the noise in the electronic circuitry is expressed in terms of equivalent turbulence intensity, what is the permissible noise level?

Note: There is another page. →

4. (30 pts) Make up your *own* order of magnitude turbulence problem. Preferably it should be something related to your research, but it could also be fun like something related to the special effects in Hollywood movies, or just something interesting in everyday life (like the kitchen mixer problem, volcanoes, fires, etc.). Write it out like a homework problem or exam problem. Provide a full solution (don't forget your shovels). Also do an analysis of how many years it would take to do a full DNS solution of this problem, like we did in class for the mixer bowl.

5. (20 pts) Consider stationary, two-dimensional (in the x-y plane), incompressible, turbulent flow over an infinite porous flat plate parallel to the x-axis. Ignore gravity. A uniform freestream velocity U_∞ flows from left to right everywhere in the flow except near the wall, where a boundary layer exists. Suction is applied uniformly everywhere along the plate through billions of tiny holes such that $V = -v_w =$ constant everywhere in the flow. The continuity and streamwise momentum equations for the mean flow reduce to $\boxed{\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} = 0}$ and $\boxed{U\frac{\partial U}{\partial x} + V\frac{\partial U}{\partial y} = -\frac{1}{\rho}\frac{\partial P}{\partial x} + \frac{\partial}{\partial y}\left(\nu\frac{\partial U}{\partial y} - \overline{uv}\right)}$.



Assume that suction velocity v_w is a constant of magnitude less than U and there is no imposed pressure gradient in the flow field, $\frac{\partial P}{\partial x} = 0$.

(**a**) Show that U must be a function of y only and that BL thickness δ is constant [neither U nor δ vary with x].

(**b**) Show that the x-momentum equation reduces to $\boxed{-v_w U = \nu\frac{dU}{dy} - \overline{uv} - \frac{\tau_w}{\rho}}$. where $\tau_w \equiv \mu\left.\frac{dU}{dy}\right)_w$ is the shear stress at the wall. *Note*: Be careful to distinguish between lower case Roman "vee" (v) [1st and 3rd terms above] and Greek letter nu (ν) [2nd term above] since these characters look very similar and sometimes lead to confusion and error.

(**c**) Except for very close to the wall, we can neglect the viscous term compared to the Reynolds stress term. Do this and also apply Prandtl's mixing length assumption, $\boxed{-\overline{uv} = \ell^2\left(\frac{dU}{dy}\right)^2 \quad \text{where} \quad \ell = \kappa y}$ and κ is a constant. Also convert U and y into inner variables, where the standard equations for inner variables are $\boxed{u^* \equiv \sqrt{\frac{\tau_w}{\rho}}}$, $\boxed{u^+ = \frac{U}{u^*}}$, and $\boxed{y^+ \equiv \frac{yu^*}{\nu}}$.

Integrate to get a final expression for u^+ as a function of y^+, κ, v_w, and u^*. *Hints*: Separate variables to do the integration, which you should be able to do *analytically*. Your result should have a term with $\ln(y^+)$ and an unknown constant which you can leave as constant C. Make sure every term in your final equation is dimensionless since u^+ is dimensionless.

6. (10 pts) Consider isotropic, incompressible, turbulent flow in Cartesian coordinates. Write out all nine components of the Reynolds stress tensor (in 3x3 matrix format) in terms of the density, ρ_0, and the turbulent kinetic energy per unit mass, q^2. Finally, write the Reynolds stress tensor in standard Cartesian tensor notation in terms of ρ_0 and q^2.