Due: In class, Wednesday **April 24, 2019** ***[Note: Wed. not Friday]***	**Name**	**PSU ID (abc1234)**

ME 522

Spring Semester, 2019

Homework Set # 10

Professor J. M. Cimbala

For instructor or TA use only:		
Problem	**Score**	**Points**
1		15
2		15
3		20
4		20
5		30
Total:		**100**

1. (15 pts) Consider a simple turbulent shear flow, in which U is the predominant mean velocity component, and U is primarily a function of y. If an eddy viscosity assumption is to be applied, one can assume that μ_e depends only on the parameters ρ, ℓ_m, and $\left|\frac{\partial U}{\partial y}\right|$.

(a) Use the Buckingham Pi dimensional analysis technique to find an expression for eddy viscosity. Show all your work. Does your result agree with that given in class for simple shear flows? Comment on the power and/or usefulness of dimensional analysis.

(b) A thought-provoking question for discussion: Why can't μ_e depend directly on U rather than on $\left|\frac{\partial U}{\partial y}\right|$?

2. (15 pts) Without going into a lot of detail, we can think of the ***energy spectrum S(k,ε)*** as a measure of how turbulent kinetic energy is distributed as a function of wavenumber. S has dimensions of {kinetic energy per unit mass} times {length}. In the so-called inertial subrange of a boundary layer, it turns out that S is a function of both wavenumber k and turbulent dissipation rate ε.

(a) Using dimensional analysis and showing all your work, derive an expression for $S(k,\varepsilon)$. Comment on the power and/or usefulness of dimensional analysis.

(b) The exponent on k from part (a) is kind of famous. Do some reading and briefly identify and discuss the famous "law" based on this exponent.

3. (20 pts) Look back at Lecture 2 class notes, where we did an example problem of a 2-D laminar jet ejecting pollution into a lake. In this homework problem, you will repeat the same problem, but with a ***turbulent*** 2-D jet instead. The similarity solution for a 2-D incompressible stationary turbulent jet is $U = U_c \operatorname{sech}^2 \eta$, where $\eta = \sigma \frac{y}{x}$, U_c is the centerline (maximum) mean speed of the jet, and σ is a constant.

(a) Generate an expression for mass flow rate $\dot{m}$ per unit width in terms of ρ, U_c, x, and σ.

(b) Generate an expression for momentum flow rate M per unit width in terms of ρ, U_c, x, and σ.

(c) Constant σ is determined from experiments to be 7.67. Generate an expression for mass flow rate $\dot{m}$ per unit width in terms of ρ, M, and x.

(d) Finally, calculate the x location where the pollutant is diluted to 1% of its initial concentration and compare to the laminar result we obtained in class. Is your result more physically realistic?

Note: There is another page. →

4. (20 pts) Consider a 2-D (in the mean), stationary, turbulent far wake. If we ignore gravity and assume that the boundary layer approximations are valid far downstream, the x-momentum equation reduces to $U_\infty \frac{\partial}{\partial x}(U_\infty - U) = -\frac{\partial}{\partial y}(-\overline{uv})$. You do not have to prove this. It is possible to generate a similarity solution for this problem by setting $f(\eta) = \frac{U_\infty - U}{U_o}$ and $g(\eta) = \frac{-\overline{uv}}{U_o^{\,2}}$, where $\eta = \frac{y}{\delta}$, δ is the half-width of the wake, defined as the distance from the wake centerline to the point in the mean velocity profile where the velocity defect is half of its maximum value U_0 at the centerline [$U_0 = U_\infty - U_{\text{centerline}}$]. Note that both δ and U_0 are functions of x only.

- **(a)** Showing all your algebra, transform the x-momentum equation given above (PDE) into a similarity **ODE** of the form $F\eta f' + Gf + g' = 0$. Write out variables F and G in terms of δ, U_0, and U_∞, along with some derivatives of δ and U_0. What conditions on F and G are necessary in order for similarity to be achieved?
- **(b)** Assume power laws for U_0 and δ, i.e., $U_0 = Ax^a$ and $\delta = Bx^b$, where A and B are constants. Try to find exponents a and b. If your algebra is correct, you will find that more information is needed. If you were to perform a control volume analysis of the far wake (you do *not* have to do this), you would find that at any x location in the far wake, the drag per unit depth is approximately $\frac{\text{drag}}{\text{unit depth}} \approx \rho U_\infty \int_{-\infty}^{\infty} (U_\infty - U)dy = \text{constant}$. Using this additional information, find exponents a and b.
- **(c)** Define a local wake Reynolds number as $\text{Re}_\delta \equiv \frac{U_0 \delta}{\nu}$. How does Re_δ vary with downstream distance x in a turbulent far wake? Is this turbulent wake expected to re-laminarize? Why or why not?

5. (30 pts) Continuation of the previous problem – turbulent far wake. The similarity form of the x-momentum equation contains two similarity functions, $f(\eta)$ and $g(\eta)$. To solve this equation for the mean velocity profile, a turbulence model must be applied. We will use a simple 2-D eddy viscosity model, $-\overline{uv} = \nu_e \frac{\partial U}{\partial y}$.

- **(a)** Plug in the eddy viscosity model, and re-write $g(\eta)$ as a function of $f(\eta)$.
- **(b)** Reduce the momentum equation as far as possible using the result of Part (a), and also using the assumed power laws $U_0 = Ax^a$ and $\delta = Bx^b$, where exponents a and b are known from the previous problem. Assume that the eddy viscosity is at most a function of η. The equation should reduce to $f'' + \frac{B^2}{2}\frac{U_\infty}{\nu_e}(\eta f' + f) + \frac{1}{\nu_e}\frac{d\nu_e}{d\eta} f' = 0$.
- **(c)** As the simplest case, assume constant eddy viscosity, i.e., at any x location in the wake, ν_e is independent of η. Let $c_1 = \frac{B^2}{2}\frac{U_\infty}{\nu_e}$ where c_1 is a constant to be found later. Reduce the above ODE to three terms.
- **(d)** Solve the ODE analytically, applying appropriate boundary conditions. You should get $f(\eta) = \exp\left(\frac{-c_1\eta^2}{2}\right)$.
- **(e)** Find constant c_1. *Hint*: By definition, δ is the wake half-thickness, defined as the y location where $(U_\infty - U) = \frac{1}{2}U_0$.
- **(f)** Finally, plot $f(\eta)$ as a line (no symbols), and compare (on the same plot) to the experimental data of Marasli (1992) as symbols (no line). Marasli's data file is available on the ME 522 course website. Is there good agreement?