Mean Kinetic Energy and Turbulent Kinetic Energy Equations

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1. Mean Kinetic Energy (MKE) Equation

Consider nearly incompressible turbulent flow (Boussinesq approximation). Manipulation of the mean continuity and momentum equations yields an equation for the ***mean kinetic energy per unit mass*** $\frac{1}{2}U_i^2$,

$$\frac{D}{Dt}\left(\tfrac{1}{2}U_i^2\right)=\frac{\partial}{\partial x_j}\left[-\frac{PU_j}{\rho_0}+2\nu U_i E_{ij}-\overline{u_i u_j}U_i\right]+\overline{u_i u_j}\frac{\partial U_i}{\partial x_j}-\frac{g}{\rho_0}\bar{\rho}U_3-2\nu E_{ij}E_{ij} \quad (1)$$

I **II** **III** **IV** **V** **VI** **VII**

where the terms are labeled and defined as follows:

- **I** ***Total rate of change*** of mean kinetic energy per unit mass following a fluid particle.
- **II** Rate of ***spatial transport*** of mean kinetic energy per unit mass by pressure work.
- **III** Rate of ***spatial transport*** of mean kinetic energy per unit mass by mean viscous stresses.
- **IV** Rate of ***spatial transport*** of mean kinetic energy per unit mass by turbulent stresses.
- **V** Rate of ***destruction*** (or production) of mean kinetic energy per unit mass into turbulence.
- **VI** Rate of ***destruction*** (or production) of mean kinetic energy per unit mass into potential energy.
- **VII** Rate of ***viscous dissipation*** of mean kinetic energy per unit mass (turns mean kinetic energy into thermal energy, i.e., heat). *This term is always negative*, indicating a *loss* of mean kinetic energy per unit mass.

Note: In Equation (1), $E_{ij}\equiv\frac{1}{2}\left(\frac{\partial U_i}{\partial x_j}+\frac{\partial U_j}{\partial x_i}\right)$ = ***mean strain rate tensor***, sometimes called S_{ij} (as in T&L). (2)

2. Turbulent Kinetic Energy (tke) Equation (also called the *tke budget*)

Manipulation of the continuity and momentum equations for the turbulent fluctuations yields an equation for the ***turbulent kinetic energy per unit mass*** $q^2=\frac{1}{2}\overline{u_k u_k}$ (*Note*: q^2 is sometimes given the notation *K* or *k* or tke),

$$\frac{D}{Dt}\left(q^2\right)=\frac{\partial}{\partial x_j}\left[-\frac{1}{\rho_0}\overline{pu_j}-\tfrac{1}{2}\overline{u_i^2 u_j}+2\nu\overline{u_i e_{ij}}\right]-\overline{u_i u_j}\frac{\partial U_i}{\partial x_j}+g\alpha\overline{wT'}-2\nu\overline{e_{ij}e_{ij}} \quad (3)$$

I **II** **III** **IV** **V** **VI** **VII**

where the terms are labeled and defined as follows:

- **I** ***Total rate of change*** of turbulent kinetic energy per unit mass following a fluid particle.
- **II** Rate of ***spatial transport*** of turbulent kinetic energy per unit mass by pressure work.
- **III** Rate of ***spatial transport*** of turbulent kinetic energy per unit mass by turbulent velocity fluctuations (convective diffusion).
- **IV** Rate of ***spatial transport*** of turbulent kinetic energy per unit mass by turbulent viscous stresses.
- **V** Rate of ***production*** (or destruction) of turbulent kinetic energy per unit mass from mean shear.
- **VI** Rate of ***production*** (or destruction) of turbulent kinetic energy per unit mass from fluctuating potential energy (buoyancy).
- **VII** Rate of ***viscous dissipation*** of turbulent kinetic energy per unit mass (turns tke into thermal energy, i.e., heat). *This term is always negative*, indicating a *loss* of turbulent kinetic energy.

Note: In Equation (3), $e_{ij}\equiv\frac{1}{2}\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right)$ = ***fluctuating strain rate tensor***, sometimes called s_{ij} (as in T&L). (4)

3. Rate of Viscous Dissipation of Turbulent Kinetic Energy per Unit Mass

The last term (**VII**) in Equation (3) is extremely important in analysis of turbulent flows. The negative of term **VII** is given a special name and symbol – the ***viscous dissipation rate of turbulent kinetic energy per unit mass***, sometimes called the ***scalar dissipation rate*** or simply the ***dissipation rate***,

$$\varepsilon\equiv 2\nu\overline{e_{ij}e_{ij}} \quad (5)$$

Since $\overline{e_{ij}e_{ij}}$ is positive definite, *the viscous dissipation rate is always a positive quantity* ($\varepsilon>0$).